PROBLEM SET 6
Math 207A, Fall 2018
Due: Fri., Nov. 16

1. Plot the bifurcation diagram and representative phases lines for the scalar ODE

$$x_t = \mu x + x^3 - x^5.$$

Identify the bifurcation points and classify them. How would the system behave if μ is increased quasi-statically from $-\infty$ to $+\infty$? When is the equilibrium $x = 0$ linearly stable but unstable to sufficiently large perturbations?

2. Consider the scalar ODE

$$x_t = \lambda + \mu x - x^2,$$

wheer $\lambda, \mu \in \mathbb{R}$ are parameters. Sketch the bifurcation diagram for the equilibria as a function of μ for fixed λ in the cases $\lambda < 0$, $\lambda = 0$, and $\lambda > 0$. Identify the bifurcation points and classify them in each case.

3. Two rigid rods of length L are connected by a torsional spring with spring constant k and are subject to a compressive force of strength F. Explain why a reasonable model for the potential energy of the system is

$$V(x) = \frac{1}{2}kx^2 - 2FL(1 - \cos x),$$

where x is the angle of the rods to the horizontal. If the rod is strongly damped with damping constant $\beta > 0$, then the ODE for its motion is $\beta x_t + V'(x) = 0$. Show that a nondimensionalized form of the ODE is

$$x_t + x - \mu \sin x = 0, \qquad \mu = \frac{2FL}{k}.$$

Sketch a bifurcation diagram for the ODE and classify the bifurcation that occurs.

4. Consider the system

$$x_t = 1 - x - \beta xy, \qquad y_t = \beta xy - (1 + \gamma)y,$$

where $\beta, \gamma > 0$ are positive parameters. Sketch the bifurcation diagram for the equilibria as a function of β and show that a bifurcation occurs at some $\beta = \beta_*(\gamma)$. What kind of bifurcation is it? Sketch typical phase planes on $\mathbb{R}^2$ (using numerical solutions if you prefer) for β close to β_* when $\beta < \beta_*$, $\beta = \beta_*$, and $\beta > \beta_*$.