# Problem set 7

Math 207A, Fall 2018

Due: Fri., Nov. 30

**1.** The normal form for a Takens-Bogdanov bifurcation is the following $2 \times 2$ system depending on two parameters $(\mu, \nu)$:

$$\dot{x} = y, \qquad \dot{y} = \mu + \nu y + x^2 + xy.$$

(a) What is the algebraic structure of the linearized system at $(x, y; \mu, \nu) = (0, 0; 0, 0)$? Find the equilibria, classify them, and determine what bifurcations of equilibria occur as $(\mu, \nu)$ vary over $(\mu, \nu) \neq (0, 0)$.

(b) Compare your results with the global phase portraits shown in Figure 8.14 of the text by Meiss.

**2.** (a) Consider a Hamiltonian system for $x, y \in \mathbb{R}^n$

$$\dot{x} = \frac{\partial H}{\partial y}, \qquad \dot{y} = -\frac{\partial H}{\partial x}$$

with Hamiltonian $H(x, y) \in \mathbb{R}$. Show that if $\lambda$ is an eigenvalue of the linearization at an equilibrium, then so is $-\lambda$. What are the possible forms of the eigenvalues for a $2 \times 2$ system? Hint. Show that the matrix of the linearization has the form $JA$ where $A$ is symmetric and

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

(b) Suppose $x, y \in \mathbb{R}$ and

$$H(x, y, \mu) = \frac{1}{2}y^2 + V(x, \mu), \qquad V(x, \mu) = \frac{1}{4}x^4 - \frac{1}{2}\mu x^2,$$

where $\mu \in \mathbb{R}$ is a parameter. Determine the equilibria of the corresponding Hamiltonian system as a function of $\mu$, classify them. Sketch the resulting phase planes and the bifurcation diagram.