

1 Dynamical systems

- Continuous and discrete dynamical systems
- Flows and iterated maps
- Orbits/trajectories
- Topological conjugacy of flows
- Invariant sets, trapping regions, and attractors
- ω -limit sets
- Hamiltonian and gradient systems
- Poincaré maps

Teschl: Chapter 6. Meiss: Chapter 4.

2 Initial value problems

- Picard existence and uniqueness theorem
- Extension theorem
- Grönwall's inequality
- Continuous dependence on initial data

Teschl: Chapter 2. Meiss: Chapter 3.

3 Linear systems

- Autonomous linear systems
- Matrix exponential
- Stable, unstable, and center subspaces for autonomous linear systems
- Fundamental matrix for nonautonomous linear systems
- Duhamel's principle for nonhomogeneous linear systems

Teschl: Chapter 3. Meiss: Chapter 2.

4 Fixed points

- Fixed points/equilibria
- Linearization of nonlinear systems at fixed points
- Hyperbolic fixed points
- Stable and unstable manifolds
- Hartman-Grobman theorem
- Lyapunov stability and asymptotic stability of fixed points
- Lyapunov and LaSalle theorems

Teschl: Chapter 6. Meiss: Chapter 4–5.

5 One-dimensional systems

- Phase lines for autonomous scalar ODEs
- Flows on the circle

Teschl: Chapter 1. Meiss: Chapter 1.

6 Two-dimensional systems

- Phase planes
- Classification of 2×2 autonomous linear systems
- Population models
- Damped and undamped conservative systems
- Limit cycles
- Poincaré-Bendixson theorem

Teschl: Chapter 7. Meiss: Chapter 6.

7 Bifurcation theory

- Equilibrium bifurcations
- Saddle node, transcritical and pitchfork bifurcations
- Center manifolds
- Hopf bifurcation
- Global homoclinic and heteroclinic bifurcations

Meiss: Chapter 8 (excluding parts we didn't cover in class)

8 Discrete dynamical systems

- Fixed points and their stability
- Bifurcations of fixed points in maps
- Period doubling bifurcations
- Hopf bifurcation for maps
- The logistic map
- Symbolic dynamics
- Homoclinic tangles

Teschl: Chapter 10–11 (excluding parts we didn't cover in class)