PROBLEM SET 1
Math 218A, Winter 2001

1. Derive the Green's function representation for the solution of Laplace's equation in the half-plane,

$$\begin{aligned} &u_{xx} + u_{yy} = 0 \qquad y > 0, \\ &u(x,0) = f(x), \end{aligned}$$

where $u(x, y)$ is assumed to decay suuficiently quickly as $y \to \infty$.

2. Find the Green's function for Laplace's equation in the unit ball in $\mathbb{R}^3$, and write out the Green's function representation of the solution of

$$\begin{aligned} &-\Delta u = f \qquad |x| < 1, \\ &u = 0 \qquad |x| = 1. \end{aligned}$$

HINT. Use the method of images.

3. A function $u \in C^2(\Omega)$ is *subharmonic* in a domain Ω if $\Delta u \geq 0$, and *superharmonic* if $\Delta u \leq 0$. Prove that if a subharmonic function attains its maximum at any point of Ω, then it is a constant function. Suppose that

$$u, v, w \in C^2(\Omega) \cap C(\overline{\Omega}),$$

where u is subharmonic, v is harmonic, and w is superharmonic. If $u \leq v \leq w$ on $\partial\Omega$, prove that $u \leq v \leq w$ on $\overline{\Omega}$.

4. Suppose that $\{u_n\}$ is a decreasing sequence of harmonic functions defined in a domain Ω such that $\{u_n(x)\}$ converges for some popint $x \in \Omega$. Prove that the sequence converges uniformly on every closed, bounded subset of Ω. HINT. Harnack's inequality.