Problem set 4

Math 218A, Winter 2001

1. Show that
$$\frac{1}{1-(x_1+x_2+\ldots+x_n)} = \sum_{\alpha\in\mathbb{Z}_+^n} \frac{|\alpha|!x^\alpha}{\alpha!}.$$

2. Let f, g be given smooth functions defined on $[0,\infty)$ such that $f(0)=g(0)$. Find the solution $u(x,t)$ in $\{(x,t) : |x| < t, t > 0\}$ of the characteristic initial value problem
$$\begin{aligned} &u_{tt} - u_{xx} = 0,\\ &u(t,t) = f(t),\\ &u(-t,t) = g(t). \end{aligned}$$

3. Compute an equation for the characteristic variety of the two-dimensional compressible Euler equations
$$\begin{aligned} &\rho_t + (\rho u)_x + (\rho v)_y = 0,\\ &u_t + uu_x + vu_y + \frac{1}{\rho}p_x = 0,\\ &v_t + uv_x + vv_y + \frac{1}{\rho}p_y = 0, \end{aligned}$$
where $p = p(\rho)$ is a given function of ρ.

4. Prove that the series for the Tychonoff solution of the heat equation that we discussed in class converges. (See e.g. F. John, *Partial Differential Equations* for help.)