Calculus

Math 21A, Fall 2015

Sample Final Problems

1. Compute the derivatives of the following functions. You do not need to simplify your answers.

(a) $\dfrac{2x+5}{\sqrt{x^4+3}}$

(b) $e^{3x+1}\ln(x^2+1)$

(c) $\tan(\cos x+\sin x)$

(d) $\sin^{-1}\left(\sqrt{2x}\right)$

2. Evaluate the following limits or say if they do not exist (using any method you want):

(a) $\displaystyle\lim_{x\to 0}\frac{1-\cos(2x)}{xe^x-x}$

(b) $\displaystyle\lim_{x\to 0}\left\{\sin x\left[\frac{1}{x}-\frac{1}{\sin(2x)}\right]\right\}$

(c) $\displaystyle\lim_{x\to\infty}\left(\ln x\, e^{-x}\right)$

(d) $\displaystyle\lim_{x\to 1}\frac{\sin x}{x}.$

3. A 15 ft ladder is leaning against a wall. If the base of the ladder is pushed toward the wall at a speed of 2 ft/sec, at what speed is the top of the ladder moving up the wall when the base of the ladder is 6ft from the wall?

4. A pile of sand in the shape of a cone whose radius is twice its height is growing at a rate of 5 cubic meters per second. How fast is its height increasing when the radius is 20 meters? Hint. The volume of a cone of radius r and height h is $V=\frac{1}{3}\pi r^2h$.

5. State the natural domain of the function

$$y=\frac{\ln x}{x^2}.$$

Sketch the graph, identify where the graph is increasing/decreasing, the local extrema, where the graph is concave up/concave down, and the inflection points.

6. A cricket ball is projected directly upward from the ground with an initial velocity of 112 ft/s. Assuming that the acceleration due to gravity is 32 ft/sec^2, derive an equation for the height $s(t)$ of the ball above the ground after t seconds. When does the ball hit the ground?

7. A one meter high fence is eight meters in front of a high wall. Find the minimum length of a ladder resting on the fence whose foot is in front of the fence and whose top reaches the wall.

8. Define a function $f(x)$ with domain $(-\infty, \infty)$ by

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ Ax + B & \text{if } 1 < x < 2 \\ -2x^2 & \text{if } x \geq 2. \end{cases}$$

(a) Determine the constants A and B so that $f(x)$ is continuous everywhere.

(b) Is this function differentiable everywhere?

(c) Sketch the graph $y = f(x)$ in that case and determine the range of f.

9. Find the equation of the tangent line to the curve

$$(x - y)^3 = x^2 - y^2 - 2$$

at the point $(2, 1)$. At which point does this tangent line cross the x-axis and at what angle?