SECTION NUMBER: E.....

CALCULUS
Math 21A, Fall 2015
Sample Midterm 1

NAME.....

SIGNATURE.....

I.D. NUMBER.....

No books, notes, or calculators.
Unless stated otherwise, show all your work and explain your answers.

Question	Points	Score
1	20	
2	30	
3	20	
4	15	
5	15	
Total	100	

1. [20%] Say if the following statements are true or false. (For this question only, you don't have to explain your answers).

(a) If $\lim_{x\to 0} f(x) = 7$, then $f(0) = 7$.

(b) If $\lim_{x\to 0} f(x) = 1$, then $f(x) > 0$ for all nonzero x that are sufficiently close to 0.

(c) If $f(x)$ is a function with domain $[0, 1]$ and $f(0) = -1$, $f(1) = 2$, then $f(x) = 0$ for some x in (0,1).

(d) If $\lim_{x\to 0^+} f(x) = 7$, then $\lim_{x\to 0} f(x^2) = 7$.

2. [30%] Evaluate the following limits or say if they do not exist:

(a) $\lim_{x\to 2} \dfrac{x^2-4}{x^2-x-2}$;

(b) $\lim_{x\to 1} \ln\left[e^x + \ln\left(3 - \dfrac{\tan(2x)}{x}\right)\right]$;

(c) $\lim_{x\to 0^+} \dfrac{\sin(\sqrt{x})}{x}$;

(d) $\lim_{x\to\infty} \dfrac{1}{x-\sqrt{x^2+x}}$.

3. [20%] Define a function $f(x)$ with domain all real numbers x by

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ \sin(\pi/x) & \text{if } 0 < x < 2, \\ x & \text{if } x \geq 2. \end{cases}$$

At what points is $f(x)$ continuous and at what points is $f(x)$ discontinuous? What kinds of discontinuity does $f(x)$ have?

4. [15%] (a) Write an expression for the slope of the tangent line to the graph $y = x^3$ at $x = 1$.

(b) Find the slope of the tangent line in (a).

5. [15%] (a) Suppose that a function $f(x)$ is defined for all x in an interval about c, except possibly at c itself. Give the precise ϵ-δ definition of

$$\lim_{x \to c} f(x) = L.$$

(b) Use the ϵ-δ definition to prove that

$$\lim_{x \to 0} (3 - 7x^2) = 3.$$