Calculus: Math 21C, Fall 2010
Sample Questions: Midterm 2

1. Determine the interval of convergence (including the endpoints) for the following power series. State explicitly for what values of x the series converges absolutely, converges conditionally, and diverges. In each case, specify the radius of convergence R and the center of the interval of convergence a.

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n}(x-1)^n; \quad \text{(b)} \sum_{n=0}^{\infty} \frac{1}{3^n+1} x^{2n}; \quad \text{(c)} \sum_{n=0}^{\infty} \frac{1}{n^2 5^n}(2x+1)^n.$$

2. Let

$$\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}, \quad \vec{v} = -\vec{i} + 2\vec{j} + 2\vec{k}.$$

Compute: (a) $|\vec{u}|$; (b) $|\vec{v}|$; (c) the angle θ between $\vec{u}$, $\vec{v}$ (you can express it as an inverse trigonometric function); (d) the projection $\text{proj}_{\vec{v}}\vec{u}$ of $\vec{u}$ in the direction of $\vec{v}$.

3. (a) Find the area of the triangle with vertices $P(-2,2,0)$, $Q(0,1,-1)$ and $R(-1,2,-2)$.
(b) Find a parametric equation for the line in which the planes $3x-6y-4z = 15$ and $6x+y-2z=5$ intersect.

4. Let

$$f(x,y) = e^{xy}\ln(y).$$

Compute the partial derivatives f_x, f_y, f_{xx}, f_{xy} and f_{yy}. (You do NOT need to simplify your answers.)

5. (a) Find the Taylor polynomial $P_4(x)$ of degree 4 centered at $x=0$ for the function $f(x) = e^{-x^2}$.
(b) Use Taylor's theorem with remainder to estimate the maximum error $|f(x) - P_4(x)|$ for $0 \le x \le 1$.
(c) Use the results of (a) and (b) to obtain an approximate value for the integral

$$\int_0^1 e^{-x^2}\,dx$$

and estimate the maximum error in your approximate value for the integral.

6. Suppose that the functions $f(x)$, $g(x)$ have the Taylor series expansions at zero, up to second degree terms, given by

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots, \qquad g(x) = b_0 + b_1 x + b_2 x^2 + \dots$$

(a) According to Taylor's theorem, how are a_0, a_1, a_2 given in terms of f and its derivatives and b_0, b_1, b_2 in terms of g and its derivative?

(b) Find the Taylor series for $h(x) = f(x)g(x)$ at zero, up to second degree terms, by multiplying the Taylor series for $f(x)$ and $g(x)$.

(c) Use the product rule to compute $h'(x)$, $h''(x)$ in terms of the derivatives of $f(x)$, $g(x)$. Show that the use of these expressions in Taylor's theorem for $h(x)$ gives the same series as the one you found in (b).