Calculus

Math 21C, Fall 2010

Sample Questions: Midterm I

1. Do the following sequences $\{a_n\}$ converge or diverge as $n \to \infty$? Give reasons for your answer. If a sequence converges, find its limit.

(a) $a_n = \dfrac{\cos n}{n}$; (b) $a_n = \dfrac{\sqrt{n}}{\ln n}$; (c) $a_n = \sqrt{n^2+1} - n$.

2. Do the following series converge absolutely, converge conditionally, or diverge? Give reasons for your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$; (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$; (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin n$

3. Determine whether each of the following series converges or diverges and explain your answer:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+4}{6n-17}$; (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-4}{5}\right)^n$; (c) $\displaystyle\sum_{n=2}^{\infty} \sqrt{\frac{n}{n^4+7}}$;

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{5^{n+1}}{(2n)!}$; (e) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln^2 n}$; (f) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{n+2\sqrt{n}}$;

(g) $\dfrac{1}{1^4} + \dfrac{1}{2^4} - \dfrac{1}{3^4} + \dfrac{1}{4^4} + \dfrac{1}{5^4} - \dfrac{1}{6^4} + \dfrac{1}{7^4} - \dfrac{1}{9^4} + \cdots$;

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$; (i) $\displaystyle\sum_{n=1}^{\infty} \left[\tan(n) - \tan(n+1)\right]$.

4. Are the following equalities true or false? Justify your answer.

(a) $$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+\frac{1}{7^2}+\dots$$
$$=1+\frac{1}{3^2}-\frac{1}{2^2}+\frac{1}{5^2}+\frac{1}{7^2}-\frac{1}{4^2}+\frac{1}{9^2}+\frac{1}{11^2}-\frac{1}{6^2}+\dots;$$

(b) $$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}+\dots$$
$$=1+\frac{1}{3}-\frac{1}{2}+\frac{1}{5}+\frac{1}{7}-\frac{1}{4}+\frac{1}{9}+\frac{1}{11}-\frac{1}{6}+\dots;$$

5. State the definition for a sequence $\{a_n\}$ to converge to a limit L. If

$$a_n=\frac{n^2+1}{n^2} \qquad \text{for } n=1,2,3,\dots$$

prove *from the definition* that

$$\lim_{n\to\infty} a_n = 1.$$

Additional question. Does the series

$$\sum_{n=2}^{\infty}\frac{1}{(\ln n)^{\ln n}}$$

converge or diverge? Justify your answer.