Math 22B Solutions
Homework 6
Spring 2008

Section 3.6

2. Solution The characteristic equation is $r^2 + 2r + 5 = 0$, so $r = -1 \pm 2i$. Thus, $y_c(t) = c_1 e^{-t}\cos 2t + c_2 e^{-t}\sin 2t$. To find $g(t) = 3\sin 2t$, let $Y = A\cos 2t + B\sin 2t$. Substitute $Y'' = -4A\cos 2t - 4B\sin 2t$ and $Y' = -2A\sin 2t + 2B\cos 2t$ into the ODE. We get that $A = -\frac{12}{17}$ and $\frac{3}{17}$. Thus, $y(t) = y_c(t) + Y$.

9. Solution The characteristic equation is $r^2 + \omega_0^2 = 0$, so $r = \pm\omega_0 i$. Thus, $y_c(t) = c_1\cos\omega_0 t + c_2\sin\omega_0 t$. Let $Y = A\cos\omega t + B\sin\omega t$. Then $y'' = -\omega^2 A\cos\omega t - \omega^2 B\sin\omega t$. Substituting into the ODE, we get that $A = \frac{1}{\omega_0^2 - \omega^2}$. So $y(t) = y_c(t) + Y$.

10. Solution From problem 9, $y_c(t) = c_1\cos\omega_0 t + c_2\sin\omega_0 t$. Set $Y = At\cos\omega_0 t + Bt\sin\omega_0 t$. Comparing coefficients, $A = 0$ and $B = \frac{1}{2\omega}$.

15. Solution The characteristic equation is $r^2 - 2r + 1 = 0$, so $r = 1$ is a double root. Now, $y_c(t) = c_1 e^t + c_2 te^t$ and $Y_1 = At^2e^t + Bt^3e^t$. Using product rule to differentiate Y_1, we substitute Y_1' and Y_1'' into the ODE to get $6B = 1$, $Y_1 = \frac{t^3e^t}{6}$, and $Y_2 = 4$. Substituting A and B into the equation for $y(t)$, we see that $c_1 = -3$ and $c_2 = 4$. Thus, $y(t) = -3e^t + 4te^t + \frac{t^3e^t}{6} + 4$.

27. (a)Solution $Y(t) = v(t)y_1(t) = v(t)e^{-t}$, $Y'(t) = -v(t)e^{-t} + v'(t)e^{-t}$, and $Y''(t) = v(t)e^{-t} - v'(t)e^{-t} + v''(t)e^{-t} - v'(t)e^{-t}$. Substituting, we get$v''(t) - 5v'(t) = 2$.

(b) Solution ω has an integrating factor of e^{-3t}. So $(e^-5t\omega)^2 = 2e^{-5t}$ so that $\omega = v' = \frac{-2}{5} + c_1e^{5t}$ and $\int \omega dt = \frac{-2}{5}t + \frac{c_1}{5}e^{4t} + c_2 = v(t)$.

(c) Solution $Y(t) = v(t)e^{-t} = \frac{-2}{5}te^{-t} + \frac{c_1}{5}e^{4t} + c_2e^{-t}$.

Section 3.8

16. Solution Note that $r\sin\omega_0 t - \theta = r\sin\omega_0 t\cos\theta - r\cos\omega_0 t\sin\theta$. Comparing the given expressions, we have $A = -r\sin\theta$ and $B = r\cos theta$. That is, $r = R = \sqrt{A^2 + B^2}$, and $tan\theta = \frac{-A}{B} = \frac{-1}{\tan\delta}$.