

ORDINARY DIFFERENTIAL EQUATIONS  
Math 22B-002, Spring 2007  
Midterm 1

NAME.....

SIGNATURE.....

I.D. NUMBER.....

*No books, notes, or calculators.  
Unless stated otherwise, show all your work.*

Question	Points	Score
1	10	
2	10	
3	20	
4	20	
5	20	
6	20	
Total	100	

1. [10%] Say if the following ODEs are linear or nonlinear. By use of the theorems given in class, what can you say about the existence and uniqueness of a solution  $y(t)$  of the ODEs with the initial condition  $y(1) = -2$ ?

(a)  $(t + 1)y' + (\cos t)y = e^t$ .

(b)  $(y + 1)y' + (\cos t)y = e^t$ .

**2.** [10%] Suppose that for certain continuous function  $p(t)$ ,  $g(t)$  and initial time  $t_0$ , the functions  $y_1(t)$ ,  $y_2(t)$  are solutions of the initial value problems

$$\begin{aligned}y_1' + p(t)y_1 &= g(t), & y_1(t_0) &= 0, \\y_2' + p(t)y_2 &= 0, & y_2(t_0) &= 1.\end{aligned}$$

If  $c_1, c_2$  are constants, show that the function

$$y(t) = c_1y_1(t) + c_2y_2(t)$$

is the solution of the initial value problem

$$y' + p(t)y = c_1g(t), \quad y(t_0) = c_2.$$

3. [20%] Suppose that  $a, b$  are constants. Find the general solution of

$$y' + ay = b.$$

If  $a > 0$ , how does the solution behave as  $t \rightarrow +\infty$ ?

4. [20%] (a) Find the solution of the initial value problem

$$ty' + 3y = \frac{1}{t}, \quad t > 0,$$
$$y(1) = y_0.$$

(b) For what initial values  $y_0$  is the solution  $y(t)$  equal to zero for some  $t > 0$ ?

5. [20%] (a) Solve the initial value problem

$$ty' + y^2 = 0,$$
$$y(1) = 1.$$

(b) What is the largest  $t$ -interval in which the solution exists?

6. [20%] Consider the ODE

$$y' = y^2 - y^4.$$

- (a) Find all equilibrium solutions.
- (b) Sketch the phase line.
- (c) Determine the stability of the equilibria you found in (a).