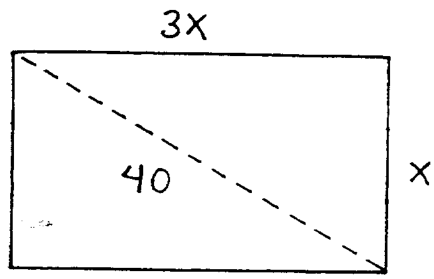
ESP
Kouba
Worksheet 1 Solutions

1.)



Pythagorean Theorem :

$$x^2 + (3x)^2 = 40^2 \rightarrow$$

$$x^2 + 9x^2 = 1600 \rightarrow$$

$$10x^2 = 1600 \rightarrow x^2 = 160 \rightarrow x \approx 12.65 \text{ cm.}$$

and $3x \approx 37.95$ cm.

2.) a.) $2x - 3 = 17$

b.) $x^2 - 100 = 0$

3.) $A = 6$

4.) $\left(x + \frac{3}{x}\right)(2x-5) = \left(x - \frac{3}{x}\right)(x+5) \rightarrow$

$$2x^2 - 5x + 6 - \frac{15}{x} = x^2 + 5x - 3 - \frac{15}{x} \rightarrow$$

$$x^2 - 10x + 9 = 0 \rightarrow x = \frac{10 \pm \sqrt{64}}{2} = 9 \text{ or } 1$$

5.) $x + \sqrt{x} = 992 \rightarrow (x - 992)^2 = (-\sqrt{x})^2 \rightarrow$

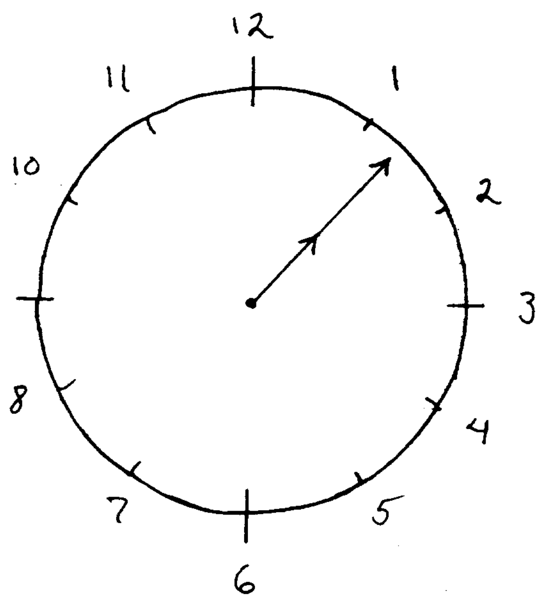
$$x^2 - 1984x + 984064 = x \rightarrow$$

$$x^2 - 1985x + 984064 = 0 \rightarrow$$

$$x = \frac{1985 \pm \sqrt{3969}}{2} = \cancel{1024} \text{ or } 961$$

NO!

6.)



after 12 noon, the hour and minute hand will meet "shortly" after 5 minutes past 1 o'clock. Determine exactly when:

Let x be minutes past 12 o'clock, then

<u>big hand</u> | <u>little hand</u>

I.) x | $5 + \frac{x}{60} \cdot 5$

$$x = 5 + \frac{x}{12} \rightarrow 12x = 60 + x$$

$$\rightarrow x = \frac{60}{11} \approx 5.45 \text{ min}$$: $\boxed{\text{5 min., 27 sec. past 1:00}}$

and II.) x | $10 + \frac{x}{60} \cdot 5$

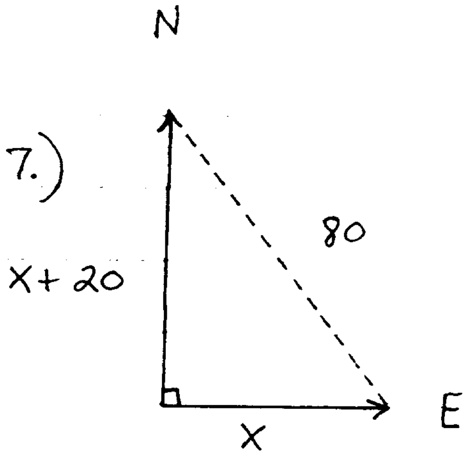
$\boxed{\text{10 min., 55 sec. past 2:00}}$

and III.) x | $15 + \frac{x}{60} \cdot 5$

$\boxed{\text{16 min., 22 sec. past 3:00}}$

and, etc. ...

7.)



Let X be Juan's speed in miles per hour. In 1 hour Juan has gone $X \cdot 1 = X$ miles, and Denise has gone $(X+20) \cdot 1 = X+20$ miles :

$$(X+20)^2 + X^2 = 80^2 \rightarrow 2X^2 + 40X - 6000 = 0 \rightarrow$$

$$X^2 + 20X - 3000 = 0 \rightarrow$$

$$X = \frac{-20 \pm \sqrt{12400}}{2} = 45.68 \text{ mph} \quad \text{or} \quad \cancel{-65.68 \text{ mph}}$$

NO

so Denise drove about

65.68 mph

8.) Let r be radius of smaller circle, then

$$\pi (10)^2 - \pi r^2 = (.75)\, \pi r^2 \rightarrow$$

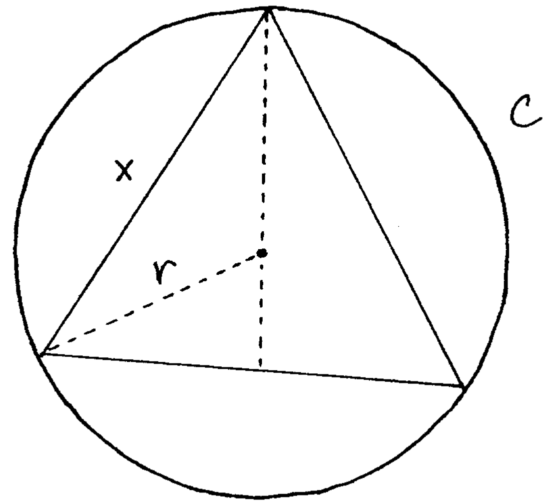
$$100 \cancel{\pi} = 1.75 \cancel{\pi} r^2 \rightarrow$$

$$r^2 = 57.14 \rightarrow \text{area} = \pi r^2 \approx \boxed{57.14\, \pi}$$

of smaller circle

9.) a.) $A = \pi r^2$

b.) $C = 2\pi r \rightarrow r = \frac{C}{2\pi}$



so $A = \pi r^2 = \pi\left(\frac{C}{2\pi}\right)^2$

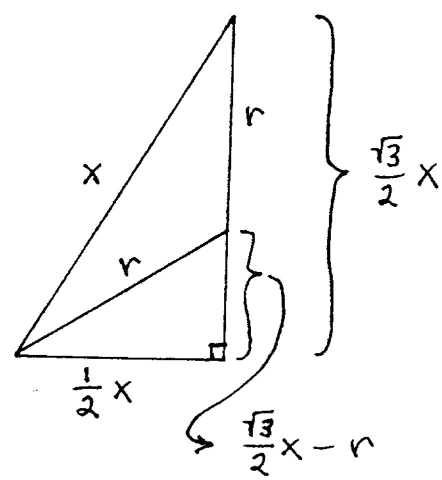
or $A = \frac{1}{4\pi} C^2$

c.) $\left(\frac{1}{2}x\right)^2 + \left(\frac{\sqrt{3}}{2}x - r\right)^2 = r^2 \rightarrow$

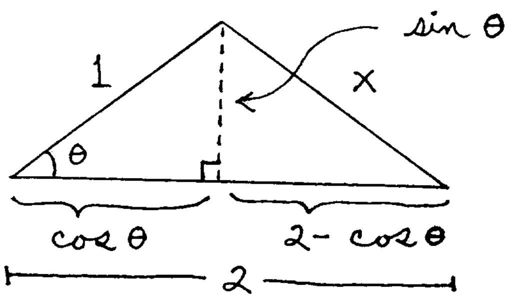
$\frac{1}{4}x^2 + \frac{3}{4}x^2 - \sqrt{3}\,rx + r^2 = r^2 \rightarrow$

$r = \frac{1}{\sqrt{3}}x$ so

$A = \pi r^2 = \pi\left(\frac{1}{\sqrt{3}}x\right)^2 = \frac{\pi}{3}x^2$



10.)



a.) $A = \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(2)(\sin\theta)$

$= \sin\theta$

b.) $A = \sin\theta$ and

$(2-\cos\theta)^2 + (\sin\theta)^2 = x^2 \rightarrow$

$4 - 4\cos\theta + (\cos^2\theta + \sin^2\theta) = x^2 \rightarrow 5 - 4\cos\theta = x^2 \rightarrow$

$\cos\theta = \frac{5-x^2}{4}$; $\cos^2\theta + \sin^2\theta = 1 \rightarrow$

$\left(\frac{5-x^2}{4}\right)^2 + (A)^2 = 1 \rightarrow A = \sqrt{1 - \left(\frac{5-x^2}{4}\right)^2}$

or $A = \frac{1}{4}\left(-x^4 + 10x^2 - 9\right)^{1/2}$.

11.) a.) $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ b.) $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

c.) $\tan\left(\frac{5\pi}{6}\right) = \dfrac{\sin\left(\frac{5\pi}{6}\right)}{\cos\left(\frac{5\pi}{6}\right)} = \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \dfrac{-1}{\sqrt{3}}$

d.) $\cot\left(\frac{4\pi}{3}\right) = \dfrac{\cos\left(\frac{4\pi}{3}\right)}{\sin\left(\frac{4\pi}{3}\right)} = \dfrac{\frac{-1}{2}}{-\frac{\sqrt{3}}{2}} = \dfrac{1}{\sqrt{3}}$

e.) $\sec\left(-\frac{7\pi}{6}\right) = \dfrac{1}{\cos\left(-\frac{7\pi}{6}\right)} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = \dfrac{-2}{\sqrt{3}}$

f.) $\csc\left(\frac{7\pi}{4} - 2\pi\right) = \csc\left(-\frac{\pi}{4}\right) = \dfrac{1}{\sin\left(\frac{-\pi}{4}\right)} = \dfrac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$

g.) Since $\sin 2\theta = 2\sin\theta\cos\theta$,

$\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right) = \frac{1}{2}\sin\left(2\cdot\frac{\pi}{8}\right) = \frac{1}{2}\sin\left(\frac{\pi}{4}\right) = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$

h.) Since $\cos 2\theta = 1 - 2\sin^2\theta$, $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$,

so $\sin\left(\frac{\pi}{8}\right) = +\sqrt{\frac{1}{2}\left(1-\cos\left(2\cdot\frac{\pi}{8}\right)\right)} = \sqrt{\frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

i.) Since $\cos 2\theta = 2\cos^2\theta - 1$, $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$,

so $\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{1}{2}\left(1+\cos\left(2\cdot\frac{\pi}{8}\right)\right)} = \sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$

j.) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ so $\tan\left(2\cdot\frac{\pi}{12}\right) = \dfrac{2\tan\left(\pi/12\right)}{1-\tan^2\left(\frac{\pi}{12}\right)} \rightarrow$

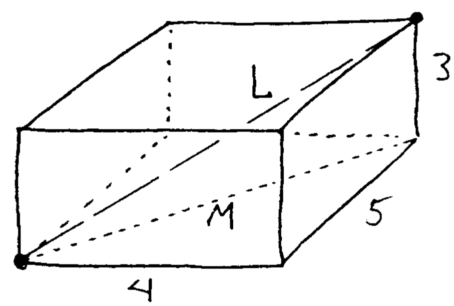
$\dfrac{1}{\sqrt{3}} = \dfrac{2\tan\left(\frac{\pi}{12}\right)}{1-\tan^2\left(\frac{\pi}{12}\right)} \rightarrow 1-\tan^2\left(\frac{\pi}{12}\right) = 2\sqrt{3}\tan\left(\frac{\pi}{12}\right) \rightarrow$

$$\tan^2\left(\frac{\pi}{12}\right) + 2\sqrt{3}\tan\left(\frac{\pi}{12}\right) - 1 = 0 \rightarrow \text{(quadratic formula)}$$

$$\tan\left(\frac{\pi}{12}\right) = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = \frac{-2\sqrt{3} \pm 4}{2} = -\sqrt{3} \pm 2$$

so $\tan\left(\frac{\pi}{12}\right) = -\sqrt{3} + 2$.

12.)



Determine L :

$4^2 + 5^2 = M^2 = 41$

and

$M^2 + 3^2 = L^2$ so that

$41 + 9 = L^2$ and $L = \sqrt{50}$ ft.

13.) $f(x) = \sqrt{x^2+9}$ so

$$f(g(x)) = \sqrt{(g(x))^2+9} = x - \sqrt{x} \rightarrow$$

$$(g(x))^2 + 9 = (x - \sqrt{x})^2 = x^2 - 2x\sqrt{x} + x \rightarrow$$

$$(g(x))^2 = x^2 - 2x\sqrt{x} + x - 9 \rightarrow$$

$$g(x) = +\sqrt{x^2 - 2x\sqrt{x} + x - 9}$$

or

$$g(x) = -\sqrt{x^2 - 2x\sqrt{x} + x - 9}$$.