

ESP  
Kouba  
Worksheet 7 Solutions

1.) a.)  $f(x) = \sin x$  is continuous for all values of  $x$ .

b.)  $f(x) = \frac{1}{\sin x}$  is continuous for all values of  $x$  except where  $\sin x = 0$ , i.e., except  $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots$ .

c.)  $f(x) = \frac{x^4 - 1}{(x-1)(x+1)}$  is continuous everywhere except  $x = 1$  and  $x = -1$ .

$$d.) \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{(x^2+1)(\cancel{x^2-1})}{(x^2-1)} = 2 = f(1)$$

$$\text{and } \lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \frac{(x^2+1)(\cancel{x^2-1})}{(x^2-1)} = 2 \neq f(-1)$$

so  $f$  is continuous everywhere except at  $x = -1$ .

$$e.) \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (x^2 + x) = 0 \quad \text{and}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{x}} = \lim_{x \rightarrow 0^+} \sqrt{x} \cdot \frac{\sin x}{\sqrt{x} \sqrt{x}}$$

$$= \lim_{x \rightarrow 0^+} \sqrt{x} \cdot \frac{\sin x}{x} = 0 \cdot 1 = 0 \quad ; \quad f \text{ is not}$$

defined at  $x = 2\pi$ , so  $f$  is continuous for all  $x$ -values except  $x = 2\pi$ .