ESP
Kouba
Worksheet 7

1.) Initially, a flock of blackbirds numbers 750 in December 1988. Two years later there are 1000 blackbirds. How many blackbirds will there be in December 1995 if the rate at which the number of birds changes is proportional to the number of birds present ?

2.) What definite integral is closely approximated by each of the following sums ?

a.) $\sum_{i=1}^{n} \frac{4+\frac{2i}{n}}{7+\sin\left(1+\frac{2i}{n}\right)} \cdot \frac{2}{n}$

b.) $\sum_{i=1}^{50} \left(\frac{i}{10}\right)^4 \cdot \left(\frac{1}{10}\right)$

c.) $\sum_{i=300}^{499} \left(\frac{i}{100}\right)^2 \cdot \left(\frac{1}{100}\right)$

d.) $\sum_{i=1501}^{5500} \left[\left(\frac{i}{500}\right)^5 + \sqrt{\frac{500}{i}}\right] \cdot \left(\frac{1}{500}\right)$

3.) a.) Let $f(x) = 3$.

i. Sketch the graph of f.

ii. Evaluate $\int_0^1 f(x)\,dx$.

b.) Let $f(x) = \begin{cases} 3 & \text{for } 0 \le x < 1 \\ 6 & \text{for } x = 1 \end{cases}$.

i. Sketch the graph of f .

ii. Evaluate $\int_0^1 f(x)\,dx$.

c.) Let f(x) = " the first digit in the decimal expansion of x " for x in [0, 1] . For example, f(0.713) = 7 and f(1/3) = 3 .

i. Sketch the graph of f.

ii. Evaluate $\int_0^1 f(x)\,dx$.

4.) Asume that y is a differentiable function of x, and compute $y' = dy/dx$ for each of the following.

a.) $y = \int_0^7 e^{x^9}\,dx$

b.) $y = \int_0^{e^x} \arctan\sqrt{t}\,dt$

c.) $y = \int_{\sin x}^{\cos x} (t^2+5)^{10}\,dt$

d.) $y^2 x + \cos(3y) = \tan^2 x$

e.) $\int_8^{e^y} [7+\cos(t^2)]\,dt = \ln y - x^5$

5.) Integrate.

a.) $\int \frac{1}{x^2}\,dx$

b.) $\int \frac{1}{1+x^2}\,dx$

c.) $\int \frac{x}{1+x^2}\,dx$

d.) $\int \frac{x^2}{1+x^2}\,dx$

e.) $\int \frac{x^3}{1+x^2}\,dx$

f.) $\int \frac{1}{4+x^2}\,dx$

g.) $\int \frac{1}{(x+1)^2}\,dx$

h.) $\int \frac{x}{(x+1)^2}\,dx$

i.) $\int \frac{x^2}{(x+1)^2}\,dx$

j.) $\int \frac{x^2}{(7x-3)^2}\,dx$

k.) $\int \frac{1}{x}\,dx$

l.) $\int \frac{\ln x}{x}\,dx$

m.) $\int \frac{(\ln x)^2}{x}\,dx$

n.) $\int \frac{1}{x \ln x}\,dx$

o.) $\int \frac{1}{x\sqrt{\ln x}}\,dx$

p.) $\int \frac{1}{x(7+\ln x)^{12}}\,dx$

q.) $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}\cdot\tan\sqrt{x}\cdot(3+\ln(\tan\sqrt{x}))}\,dx$

r.) $\int x \ln x\,dx$

s.) $\int \sqrt{x}\cdot\ln(x^3)\,dx$

t.) $\int \ln\sqrt{x}\,dx$

u.) $\int (\ln x)^2\,dx$

v.) $\int x e^{-x}\,dx$

w.) $\int \arctan x\,dx$

x.) $\int \sin\sqrt{x}\,dx$

y.) $\int e^{\sqrt{x}}\,dx$

z.) $\int x\sqrt{x+1}\,dx$

6.) Let S(x) be the square of the distance from (0, 0) to the point (x, y) on the graph of $y = e^x$. Compute the average value of S for x in [0, 2].