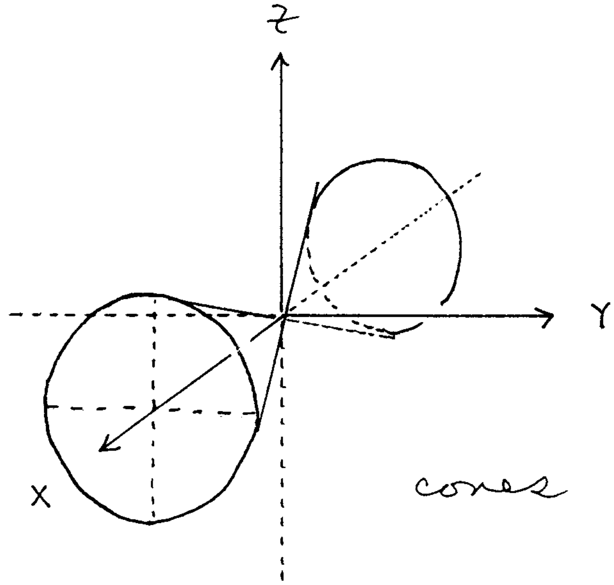
ESP
Kouba
Worksheet 3 Solutions

1.) a.) $x^2 = y^2 + z^2$

x	curve
-3	$9 = y^2 + z^2$
-2	$4 = y^2 + z^2$
-1	$1 = y^2 + z^2$
0	$0 = y^2 + z^2$
1	$1 = y^2 + z^2$
2	$4 = y^2 + z^2$
3	$9 = y^2 + z^2$



cones

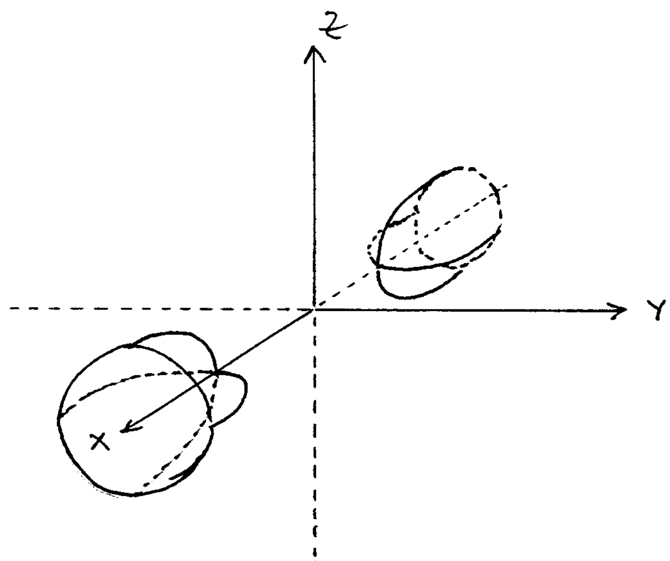
b.) $x^2 - y^2 - z^2 = 1$

$x = 0: \; -y^2 - z^2 = 1$ No!

$y = 0: \; x^2 - z^2 = 1$

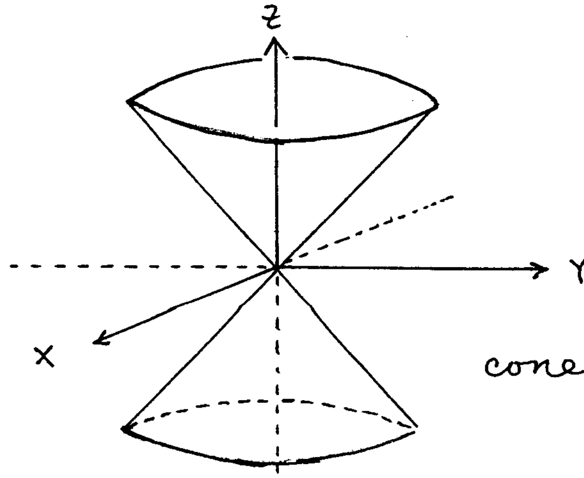
$z = 0: \; x^2 - y^2 = 1$

hyperboloid of two sheets



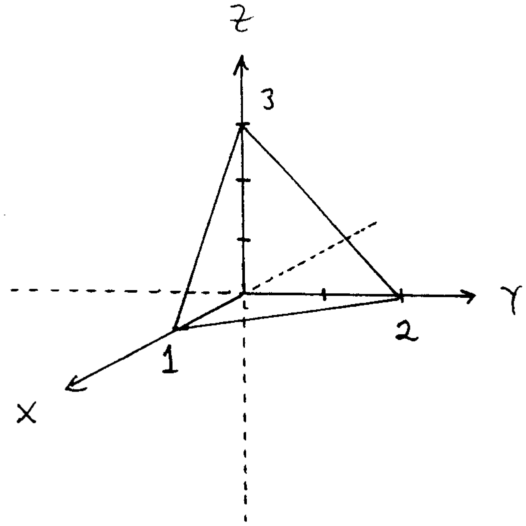
c.) $x^2 + y^2 = z^2$

z	level curves
-3	$x^2 + y^2 = 9$
-2	$x^2 + y^2 = 4$
-1	$x^2 + y^2 = 1$
0	$x^2 + y^2 = 0$
1	$x^2 + y^2 = 1$
2	$x^2 + y^2 = 4$
3	$x^2 + y^2 = 9$



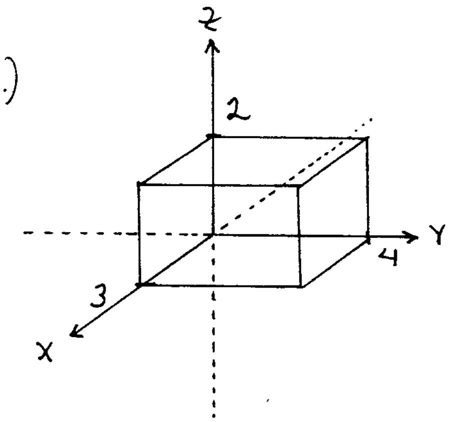
cones

d.)

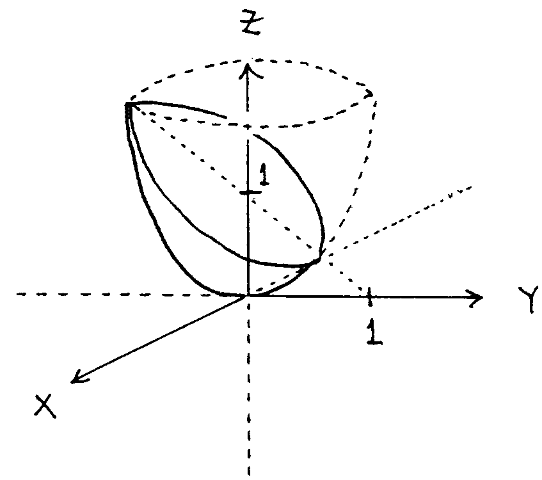


plane

2.) a.)

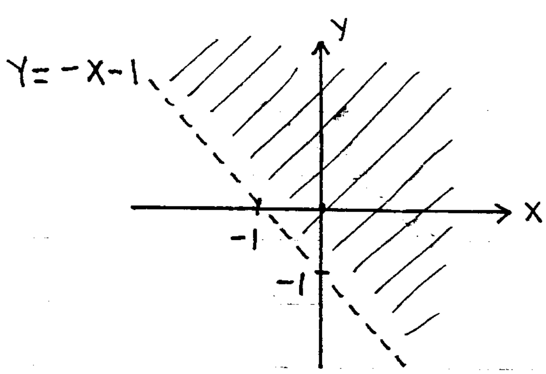


b.)



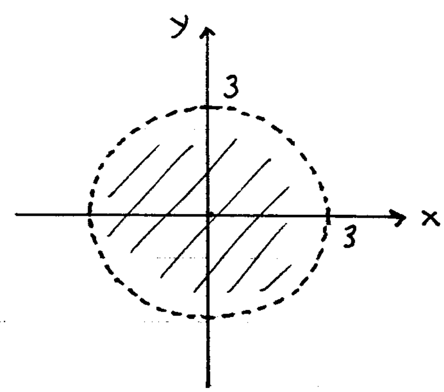
3.) a.) $1+X+Y>0$

or $Y>-X-1$

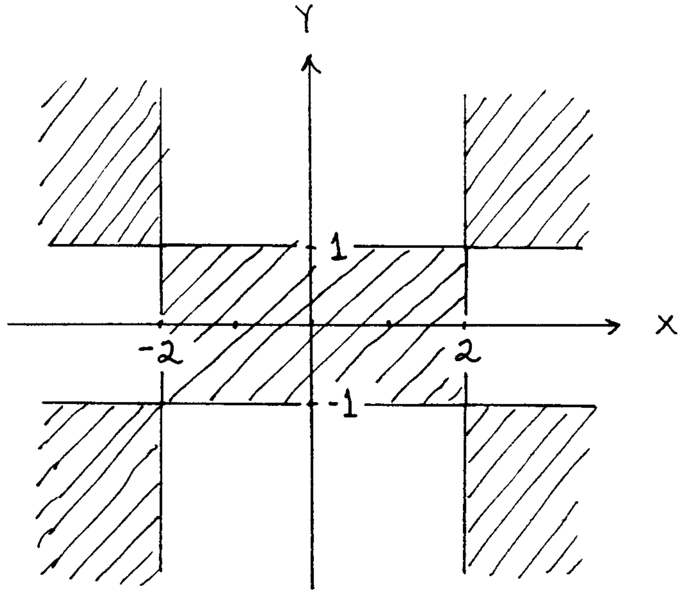


b.) $9-X^2-Y^2>0$

or $X^2+Y^2<9$



c.) $(X^2-4)(Y^2-1)=(X-2)(X+2)(Y-1)(Y+1)\geq 0$



4.) a.) $\Delta f = f(2.1, 3.3) - f(2,3) = 2.553$ and

$$df = f_x(2,3) \cdot \Delta x + f_y(2,3) \cdot \Delta y$$
$$= 2(2)(3) \cdot (0.1) + (2)^2 \cdot (0.3) = 2.4$$

b.) $\Delta f = f(1.01, 1.98) - f(1,2) = -0.03818$ and

$$df = f_x(1,2) \cdot \Delta x + f_y(1,2) \cdot \Delta y$$
$$= \tfrac{1}{2}(9)^{-1/2} \cdot (0.01) + \tfrac{1}{2}(9)^{-1/2} \cdot 3(2)^2 \cdot (-0.02) = -0.03833$$

c.) $\Delta f = f(\ln 1.01, 2.9) - f(0,3) = -0.12871$ and

$$df = f_x(0,3) \cdot \Delta x + f_y(0,3) \cdot \Delta y$$
$$= -\frac{3}{e^0} \cdot (\ln 1.01) + \frac{1}{e^0}(-0.1) = -0.12985$$

5.) $\frac{|\Delta r|}{r} \le 0.02$ and $\frac{|\Delta h|}{h} \le 0.03$

a.) $V = \pi r^2 h$ so

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V_r \cdot \Delta r + V_h \cdot \Delta h|}{V}$$

$$= \frac{|2\pi r h \cdot \Delta r + \pi r^2 \cdot \Delta h|}{\pi r^2 h} \le 2\frac{|\Delta r|}{r} + \frac{|\Delta h|}{h}$$

$$= 2(0.02) + (0.03) = 7\% .$$

b.) $S = 2\pi r^2 + 2\pi r h$ so

$$\frac{|\Delta S|}{S} \approx \frac{|dS|}{S} = \frac{|S_r \cdot \Delta r + S_h \cdot \Delta h|}{S}$$

$$= \frac{|(4\pi r + 2\pi h)\cdot \Delta r + (2\pi r)\cdot \Delta h|}{2\pi r^2 + 2\pi r h}$$

$$\le \frac{(2r+h)\cdot \frac{|\Delta r|}{r} \cdot \cancel{r} + (\cancel{r}) \cdot \frac{|\Delta h|}{h} \cdot h}{r^{\cancel{2}} + \cancel{r}h}$$

$$\le \frac{(2r+h)\cdot(0.02) + h\,(0.03)}{r+h} = \frac{(0.04)r + (0.05)h}{r+h}$$

$$\le \frac{(0.05)\,r + (0.05)h}{r+h} = 5\% .$$

6.) $\frac{|\Delta l|}{l} \le 0.01$, $\frac{|\Delta w|}{w} \le 0.02$, $\frac{|\Delta h|}{h} \le 0.04$

a.) $V = lwh$ so

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V_l \cdot \Delta l + V_w \cdot \Delta w + V_h \cdot \Delta h|}{V}$$

$$= \frac{|wh \cdot \Delta l + lh \cdot \Delta w + lw \cdot \Delta h|}{lwh}$$

$$\le \frac{|\Delta l|}{l} + \frac{|\Delta w|}{w} + \frac{|\Delta h|}{h} \le (0.01) + (0.02) + (0.04) = 7\%$$

b.) $S = 2lw + 2lh + 2wh$ so

$$\frac{|\Delta S|}{S} \approx \frac{|dS|}{S} = \frac{|S_l \cdot \Delta l + S_w \cdot \Delta w + S_h \cdot \Delta h|}{S}$$

$$= \frac{|(\cancel{2}w + \cancel{2}h) \cdot \Delta l + (\cancel{2}l + \cancel{2}h) \cdot \Delta w + (\cancel{2}l + \cancel{2}w) \cdot \Delta h|}{\cancel{2}lw + \cancel{2}lh + \cancel{2}wh}$$

$$\le \frac{(w+h) \cdot \frac{|\Delta l|}{l} \cdot l + (l+h) \cdot \frac{|\Delta w|}{w} \cdot w + (l+w) \cdot \frac{|\Delta h|}{h} \cdot h}{lw + lh + wh}$$

$$\le \frac{(w+h)(0.01) \cdot l + (l+h)(0.02) \cdot w + (l+w) \cdot (0.04) \cdot h}{lw + lh + wh}$$

$$= \frac{lw(0.03) + lh(0.05) + wh(0.06)}{lw + lh + wh}$$

$$\le \frac{lw(0.06) + lh(0.06) + wh(0.06)}{lw + lh + wh} = 6\%$$