Math 16B
Kouba
Discrete Probability

<u>Example</u>: Consider all Girl/Boy gender possibilities by age in a family of 3 children.

a.) List the <u>sample</u> <u>space</u>, S:

	1	2	3
S:	G	G	G
	B	G	G
	G	B	G
	G	G	B
	B	B	G
	B	G	B
	G	B	B
	B	B	B

assume that Boys and Girls are equally likely.

Definition: Let S be a sample space. A **random variable** is a function X that assigns a number to each outcome in S.

b.) Let random variable X be the # of girls in each family. List all values of X:

X (# of girls)
0
1
2
3

Definition: The **probability** of a random variable X is

$$P(X) = \frac{\text{Frequency of } X \text{ in } S}{\text{Total \# of Outcomes in } S}.$$

x	$P(x)$
0	$P(x=0) = 1/8 = 0.125 = 12.5\%$
1	$P(x=1) = 3/8 = 0.375 = 37.5\%$
2	$P(x=2) = 3/8 = 0.375 = 37.5\%$
3	$P(x=3) = 1/8 = 0.125 = 12.5\%$

Remark : 1.) $0 \leq P(x) \leq 1$

2.) The sum of all probabilities for a random variable x is

$$1 = 1.00 = 100\% .$$

Compute the following probabilities for this example:

1.) $P(\text{exactly 3 girls}) = \frac{1}{8} = 12.5\%$

2.) $P(\text{1 or 2 girls}) = \frac{6}{8} = \frac{3}{4} = 75\%$

3.) $P(\text{at least 2 boys}) = \frac{4}{8} = \frac{1}{2} = 50\%$

4.) $P(\text{less than 3 boys}) = \frac{7}{8} = 87.5\%$

5.) $P(\text{at most 1 girl}) = \frac{4}{8} = 50\%$

6.) $P(\text{4 boys}) = \frac{0}{8} = 0 = 0\%$

7.) $P(\text{0, 1, 2, or 3 girls}) = \frac{8}{8} = 1 = 100\%$

Expected Value (Mean), Variance, and Standard Deviation

(SEE Handout)

Compute the Expected Value, Variance, and Standard Deviation for this random variable X :

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Discrete Probability , Definitions

Let $\{ x_1 , x_2 , x_3 , \ldots , x_m \}$ be m outcomes for a discrete random variable, x , and let $p_1 , p_2 , p_3 , \ldots , p_m$ be the associated probabilities.

1.) The <u>MEAN</u> , μ , or <u>EXPECTED VALUE</u> , $E(x)$, of x is

$$\mu = E(x) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \cdots + x_m p_m \; .$$

2.) The <u>VARIANCE</u> , $V(x)$, of x is

$$V(x) = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + (x_3 - \mu)^2 p_3 + \cdots + (x_m - \mu)^2 p_m \; .$$

3.) The <u>STANDARD DEVIATION</u> , σ , of x is $\sigma = \sqrt{V(x)}$.

1.) $\mu = E(x) = (0)\left(\frac{1}{8}\right) + (1)\left(\frac{3}{8}\right) + (2)\left(\frac{3}{8}\right)$
$+ (3)\left(\frac{1}{8}\right) = \frac{12}{8} = 1.5$ girls

2.) $V(x) = \left(0 - \frac{3}{2}\right)^2\left(\frac{1}{8}\right) + \left(1 - \frac{3}{2}\right)^2\left(\frac{3}{8}\right)$
$+ \left(2 - \frac{3}{2}\right)^2\left(\frac{3}{8}\right) + \left(3 - \frac{3}{2}\right)^2\left(\frac{1}{8}\right)$
$= \left(\frac{9}{4}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{8}\right) + \left(\frac{9}{4}\right)\left(\frac{1}{8}\right)$
$= \frac{24}{32} = \frac{3}{4} = 0.75$ girls^2

3.) $\sigma = \sqrt{V(x)} = \sqrt{0.75} \approx 0.87$ girls