Math 16B
Kouba
Supplemental Algebra

SA1.) For tax purposes you must report the value of a refrigerator which you bought three years ago for $1250. Assuming a straight line depreciation in value for 10 years, that is, the refrigerator is worth $1250 when you buy it and is worth $0 after ten years, what is the current value of your refrigerator ?

SA2.) In the early 1920's, Germany had tremendously high inflation, called hyperinflation. If a loaf of bread cost 1/4 DM (Deutschemark) in 1919 and 2,400,000 DM in 1922, what was the average yearly inflation rate between 1919 and 1922 ?

SA3.) Each planet moves around the sun in an elliptical orbit. The orbital period, T, of a planet is the time it takes the planet to go once around the sun. The semimajor axis, S, of each planet's orbit is the average of the largest and the smallest distances between the planet and the sun. Kepler discovered that the period of a planet is proportional to the 3/2 power of its semimajor axis.

Write T as a function of S using the facts that the semimajor axis of the earth is 150 million km and its orbital period is 365 days.

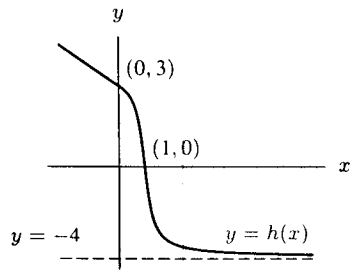
a.) What is the orbiting period (in days) of Mercury, the closest planet to the sun, with a semimajor axis of 58 million km ?

b.) What is the period (in years) of Pluto, the farthest planet from the sun, with a semimajor axis of 6000 million km ?

SA4.) Consider the graph of y = h(x) in the figure at right. Sketch a graph of

a.) $y = h^{-1}(x)$, the inverse of h.

b.) $y = 1/h(x)$, the reciprocal of h.



SA5.) A *catalyst* in a chemical reaction is a substance which speeds up the reaction but which does not itself change. If the product of a reaction is itself a catalyst, the reaction is said to be *autocatalytic*. Suppose the rate, r, of a particular autocatalytic reaction is proportional to the quantity of the original material remaining times the quantity of product, p, produced. If the initial quantity of the original material is A and the amount remaining is A - p :

a.) Express r as a function of p.
b.) What is the value of p when the reaction is proceeding fastest ?

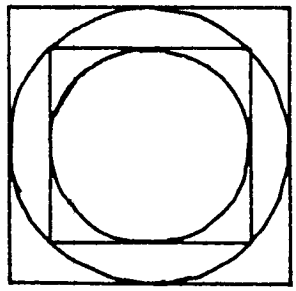
SA6.) Find the inverse of each of the following functions.

a.) $f(x) = 7\,e^{x^3+1} - 2$
b.) $f(x) = 3\,\log_7(4x-5)$
c.) $f(x) = \ln(x+1) - \ln(x+2)$

SA7.) Sketch the following graphs and find all points of intersection.

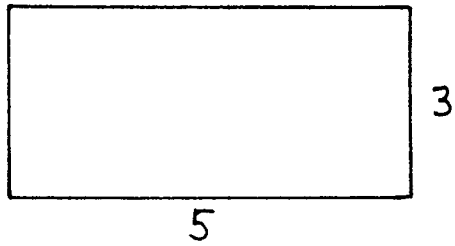
a.) $y = x,\ y = 2 - x^2$
b.) $y = 0,\ y = x^2 - 3x - 4$
c.) $y = x^2 - 4x + 3,\ y = -x^2 + 2x + 3$
d.) $y = x - 1,\ y = (x-1)^3$
e.) $y = -x^2 + 2x,\ y = 3x^3 - x^2 - 10x$
f.) $x = y^2,\ x = y + 2$
g.) $x = y(2-y),\ x = -y$
h.) $y = \sqrt{x},\ y = (1/4)x$
i.) $y = x^{1/3},\ y = (1/9)x$
j.) $y = \sin x,\ y = \sin 2x$ on $[0, 2\pi]$

SA8.) Consider the given figure of nested squares and circles. If the perimeter of the larger square is 8, what is the area of the smaller circle ?



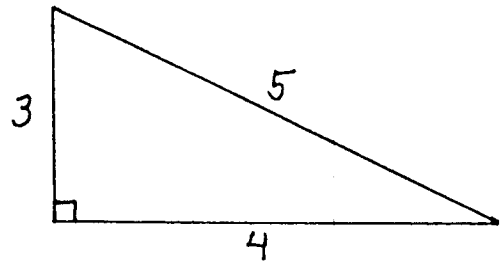
SA9.) Consider the given 3 by 5 rectangle. Find the volume ($V = \pi r^2 h$) of the imaginary cylinder formed by revolving the rectangle around an

a.) edge of length 3.
b.) edge of length 5.



SA10.) Consider the given right triangle. Find the volume of the imaginary solid (Volume of cone is $V = (1/3)\pi r^2 h$.) formed by revolving the triangle around its

a.) edge of length 3.
b.) edge of length 4.
c.) edge of length 5.



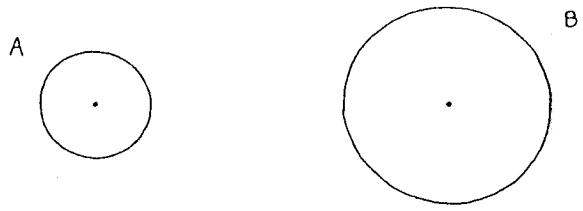
SA11.) Consider the region bounded by the graphs of $y = 0$ and $y = \sqrt{1-(x-1)^2}$. Find the volume of the imaginary solid formed by revolving the region about the x-axis. HINT : The volume of a sphere is $V = (4/3)\,\pi\, r^3$.

SA12.) Wheel A has a radius of 2 inches. Wheel B has a radius of 6 inches. See Fig. 1

a. If wheel A and wheel B begin rolling on a flat surface and each completes exactly ten rotations, what is the total distance (inches) traveled by each wheel ?

b. How many rotations are required for each wheel to travel exactly 10 feet ?

Fig. 1



SA13.) Sprocket A has a radius of 2 inches and is firmly attached to a wheel of radius 6 inches. Sprocket B has a radius of 6 inches and is firmly attached to a wheel of radius 30 inches. Sprockets A and B are connected with a closed belt about their circumferences. See Fig. 2

a. If sprocket A rotates exactly 3 times, how many times will sprocket B rotate ?

b. If sprocket B rotates exactly 3 times, how many times will sprocket A rotate ?

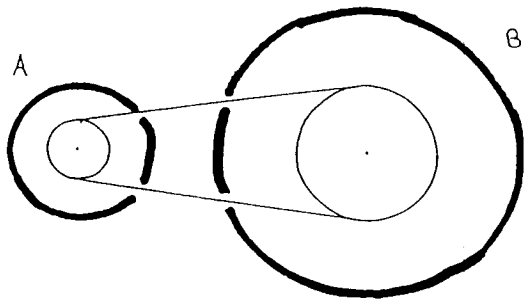
c. If sprocket A rotates exactly 3 times per second, how fast (in./sec. and m.p.h.) is a fixed point on the circumference of

i. sprocket A traveling ? ii. wheel B traveling ?

d. If sprocket B rotates exactly 3 times per second, how fast (in./sec. and m.p.h.) is a fixed point on the circumference of

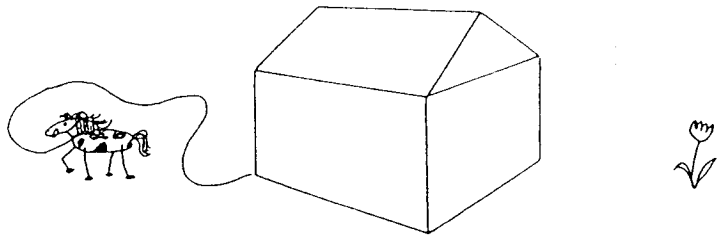
i. sprocket B traveling ? ii. wheel A traveling ?

Fig. 2



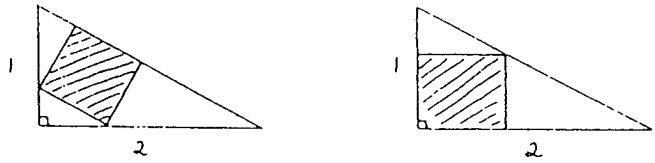
SA14.) A horse is tethered by a rope to the corner of a shed with a 10 ft. by 10 ft. floor (Fig. 3). If the rope is 40 ft. long, draw a "bird's eye view" of the shape of the horse's grazing area. How close can you plant flowers to the shed and keep the horse from eating them ?

Fig. 3



SA15.) Each of the following right triangles has a shaded square inscribed in its interior (Fig. 4). Which square has the larger area ?

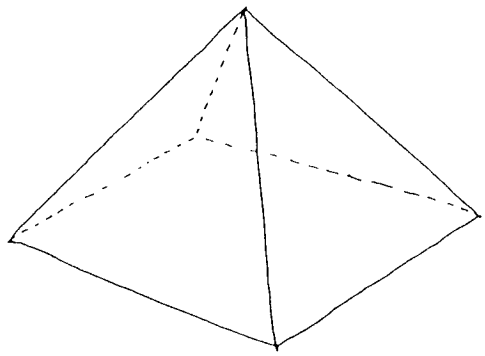
Fig. 4



SA16.) Consider the given symmetrical 4-sided pyramid of height 3 ft. and with square base of area 16 ft. ^{2} (Fig. 5).

a. Compute the total surface area of the pyramid.
b. Find the radius of the largest sphere which can be inscribed inside the pyramid.

Fig. 5



SA17.) Solve for the unknown variables in the given system of equations.

a.) $A + B = 3$ and $2A - 3B = 4$

b.) $2A - 5B = 3$ and $3A + 2B = 7$

c.) $100 = C e^{k}$ and $200 = C e^{4k}$

d.) $16 = C e^{5k}$ and $90 = C e^{20k}$

e.) $50 = 7 C e^{3k+1}$ and $203 = 5 C e^{8k+11}$