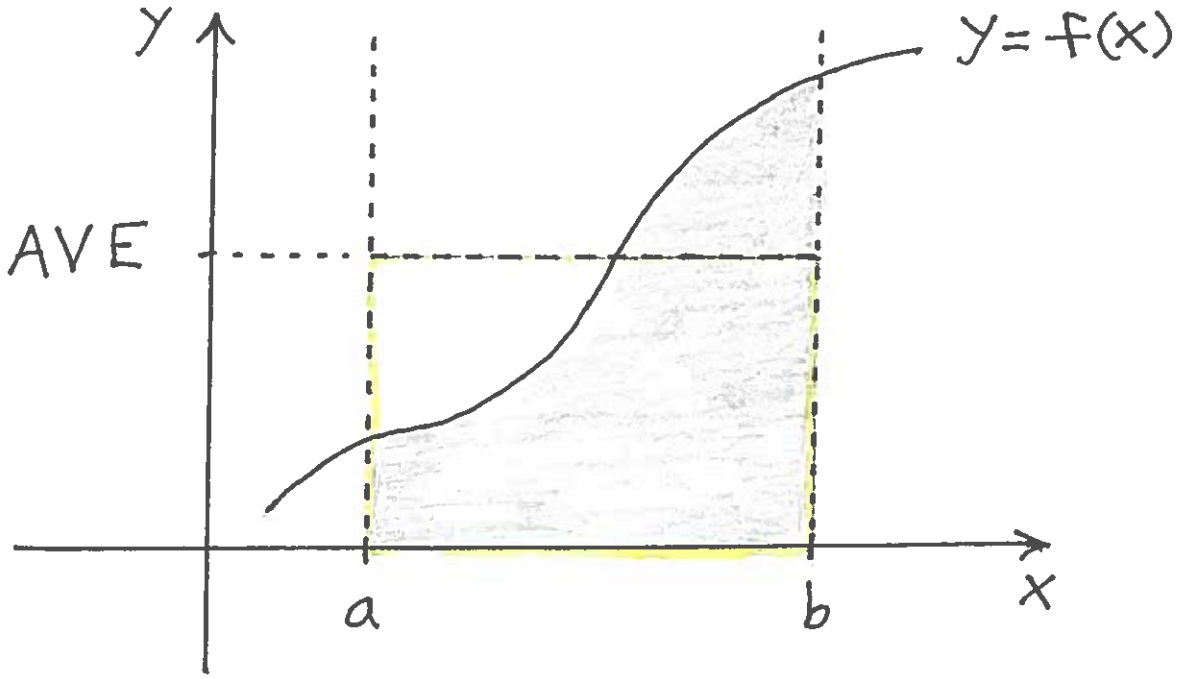
Average Value of a Function

TASK: Find the rectangle whose AREA is equal to the AREA of the shaded region below the graph of $y = f(x)$ on the interval $[a, b]$:



Then

$$(AVE)(b-a) = \int_a^b f(x)\,dx \longrightarrow$$

$$AVE = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Definition: The **average Value** of function $y = f(x)$ on interval $[a, b]$ is

$$AVE = \frac{1}{b-a} \int_a^b f(x)\,dx$$

Example: Find the average value of each function on the given interval.

1.) $y = x^3$

a.) $[0, 4]$

b.) $[1, 2]$

c.) $[-3, 3]$

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a.) $AVE = \frac{1}{4-0} \int_0^4 x^3\,dx$

$= \frac{1}{4} \cdot \frac{1}{4} x^4 \Big|_0^4 = \frac{1}{16}(4)^4 = 16$

b.) $AVE = \frac{1}{2-1}\int_1^2 x^3\,dx$

$= (1)\cdot\frac{1}{4}x^4\Big|_1^2 = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$

c.) $AVE = \frac{1}{3-(-3)}\int_{-3}^{3} x^3\,dx$

$= \frac{1}{6}\cdot\frac{1}{4}x^4\Big|_{-3}^{3} = \frac{1}{24}\left(\frac{81}{4} - \frac{81}{4}\right)$

$= \frac{1}{24}(0) = 0$

2.) $y = e^x$

a.) $[0, 1]$

b.) $[0, \ln 3]$

c.) $[\ln(\frac{1}{2}), \ln 2]$

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a.) $AVE = \frac{1}{1-0}\int_0^1 e^x\,dx$

$= e^x\Big|_0^1 = e^1 - e^0 = e - 1$

b.) $AVE = \frac{1}{\ln 3 - 0}\int_0^{\ln 3} e^x\,dx$

1.) What is the room's temperature for a.) $t=0$? b.) $t=4$? c.) $t=16$?

2.) What is the room's AVERAGE temperature for $0 \le t \le 16$?

1.) a.) $t=0$: $T = 40 + 8\sqrt{0} = 40^\circ F$

b.) $t=4$: $T = 40 + 8\sqrt{4} = 40 + 16 = 56^\circ F$

c.) $t=16$: $T = 40 + 8\sqrt{16} = 40 + 32 = 72^\circ F$

2.) $AVE = \frac{1}{16-0}\int_0^{16} (40 + 8\sqrt{t})\,dt$

$= \frac{1}{16}\left(40t + 8 \cdot \frac{2}{3}t^{3/2}\right)\Big|_0^{16}$

$= \frac{1}{16}\left(40(16) + \frac{16}{3}(16)^{3/2}\right) - \frac{1}{16}\left(40(0) + \frac{16}{3}(0)^{3/2}\right)$ (0)

$= \frac{1}{16}\left(640 + \frac{16}{3}(64)\right) = \frac{1}{16}\left(\frac{1920}{3} + \frac{1024}{3}\right)$

$= \frac{1}{16}\left(\frac{2944}{3}\right) = \frac{184}{3} \approx 61.33^\circ F$

$$= \frac{1}{\ln 3} e^x \Big|_0^{\ln 3} = \frac{1}{\ln 3}(e^{\ln 3} - e^0)$$

$$= \frac{1}{\ln 3}(3-1) = \frac{2}{\ln 3}$$

c.) $$AVE = \frac{1}{\ln 2 - \ln(\frac{1}{2})} \int_{\ln(\frac{1}{2})}^{\ln 2} e^x \, dx$$

$$= \frac{1}{\ln 2 - (\underset{0}{\ln 1} - \ln 2)} e^x \Big|_{\ln(1/2)}^{\ln 2}$$

$$= \frac{1}{2\ln 2}(e^{\ln 2} - e^{\ln(1/2)})$$

$$= \frac{1}{2\ln 2}(2 - \frac{1}{2}) = \frac{1}{2\ln 2}(\frac{3}{2}) = \frac{3}{4\ln 2}$$

Example: assume that the air temperature T (°F) in a closed room after t minutes is given by

$$T = 40 + 8\sqrt{t}$$

for $0 \le t \le 16$.