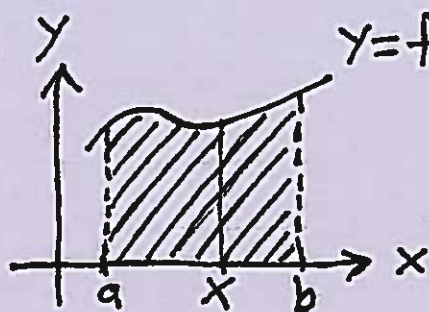


Elementary
Applications
of
Definite
Integrals

Applications of the Definite Integral

I.)

Recall:

$$\text{Area} = \int_a^b f(x) dx$$

$\uparrow$ height $\uparrow$ width

II.) If $f(t)$ is velocity ($\frac{\text{distance}}{\text{time}}$), then distance traveled for $t=a$ to $t=b$ is

$$\text{Distance} = \int_a^b f(t) dt$$

$\uparrow$ distance $\uparrow$ time
 time

III.) If $f(x)$ is the density ($\frac{\text{mass}}{\text{length}}$) of a wire from $x=a$ to $x=b$, then the total mass of the wire is

$$\text{Mass} = \int_a^b f(x) dx$$

$\uparrow$ mass $\uparrow$ length
 length

IV. If $A(x)$ is the cross-sectional area of a 3D-solid from $x=a$ to $x=b$, then the volume of the solid is

$$\text{Volume} = \int_a^b A(x) dx$$

$\uparrow$ area $\uparrow$ length

Example : Water is leaking from a faucet at the rate of $\sqrt{t}$ gal./hr. for $0 \leq t \leq 4$; t is hours.

1.) How fast is the water leaking when $t=1$ hr.?
 $t=4$ hrs.?

2.) What is the total amount of water lost to leakage for $0 \leq t \leq 4$?

1.) a.) $t=1$: $\sqrt{1} = 1$ gal./hr.

b.) $t=4$: $\sqrt{4} = 2$ gal./hr.

2.) Total = $\int_0^4 \sqrt{t} dt$

$$= \frac{2}{3} t^{3/2} \Big|_0^4$$

$$= \frac{2}{3} (4)^{3/2} - \frac{2}{3} (0)^{3/2}$$

$$= \frac{2}{3} (8) - 0 = \frac{16}{3} \text{ gal.}$$

Example: Snow is falling at the rate of $\frac{3}{t+1}$ in./hr. for $0 \leq t \leq 9$; t is hours.

1.) at what rate is snow falling when $t=0$ hrs.?
 $t=5$ hrs.?

2.) What is the total amount of snowfall for $0 \leq t \leq 9$?

1.) a.) $t=0$: $\frac{3}{(0)+1} = 3$ in./hr.

b.) $t=5$: $\frac{3}{(5)+1} = \frac{1}{2}$ in./hr.

2.) Total = $\int_0^9 \frac{3}{t+1} dt$
 $= 3 \ln|t+1| \Big|_0^9$
 $= 3 \ln 10 - 3 \ln 1$
 $= 3 \ln 10 \approx 6.91$ in.