Math 16B
Section 8.5

Antiderivatives of Trig Functions

SEE FORMULAS on the next page.

Example: Determine the following antiderivatives.

1.) $\int (\cos x + \sec^2 x)\, dx$

$= \sin x + \tan x + C$

2.) $\int \sin 3x \, dx$

(Let $u = 3x \xrightarrow{D} du = 3\,dx \rightarrow \frac{1}{3}du = dx$)

$= \frac{1}{3}\int \sin u \, du = \frac{1}{3} \cdot -\cos u + C$

$= \frac{-1}{3}\cos 3x + C$

3.) $\int \sec 4x \, dx$

(Let $u = 4x \xrightarrow{D} du = 4\,dx \rightarrow \frac{1}{4}du = dx$)

$= \frac{1}{4}\int \sec u \, du = \frac{1}{4}\ln|\sec u + \tan u| + C$

$= \frac{1}{4}\ln|\sec 4x + \tan 4x| + C$

4.) $\int \sin^3 x \cos x \, dx$

(Let $u = \sin x \xrightarrow{D} du = \cos x \, dx$)

Math 16 B
Kouba
Trigonometry Formulae

1.) $D(\sin x) = \cos x$

2.) $D(\cos x) = -\sin x$

3.) $D(\tan x) = \sec^2 x$

4.) $D(\sec x) = \sec x \tan x$

5.) $D(\cot x) = -\csc^2 x$

6.) $D(\csc x) = -\csc x \cot x$

1.) $\int \cos x \, dx = \sin x + C$

2.) $\int \sin x \, dx = -\cos x + C$

3.) $\int \tan x \, dx = \ln|\sec x| + C$

4.) $\int \sec x \tan x \, dx = \sec x + C$

5.) $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

6.) $\int \sec^2 x \, dx = \tan x + C$

7.) $\int \cot x \, dx = \ln|\sin x| + C$

8.) $\int \csc x \cot x \, dx = -\csc x + C$

9.) $\int \csc x \, dx = \ln|\csc x - \cot x| + C$

10.) $\int \csc^2 x \, dx = -\cot x + C$

1.) $\cos^2 x + \sin^2 x = 1$

2.) $1 + \tan^2 x = \sec^2 x$

3.) $1 + \cot^2 x = \csc^2 x$

4.) $\sin 2x = 2 \sin x \cos x$

5.) $\cos 2x = 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$
$= \cos^2 x - \sin^2 x$

TRIG IDENTITIES

5.) $\int \tan x \cdot \sec^2 x \, dx$

(Let $u = \tan x \xrightarrow{D} du = \sec^2 x \, dx$)

$= \int u \, du = \frac{1}{2}u^2 + C = \frac{1}{2}(\tan x)^2 + C$

6.) $\int \sin x (1 + \cos^2 x) \, dx$

$= \int (\sin x + \sin x \cos^2 x) \, dx$

$= -\cos x + \int \sin x \cos^2 x \, dx$

(Let $u = \cos x \xrightarrow{D} du = -\sin x \, dx$

$\rightarrow -du = \sin x \, dx$)

$= -\cos x + -\int u^2 \, du$

$= -\cos x - \frac{1}{3}u^3 + C$

$= -\cos x - \frac{1}{3}(\cos x)^3 + C$

7.) $\int (\sin x - \cos x)^2 \, dx$

$= \int (\sin^2 x - 2\sin x \cos x + \cos^2 x) \, dx$

$= \int [(\sin^2 x + \cos^2 x) - 2\sin x \cos x] \, dx$

(Use Trig Identity 1.)

$= \int [1 - 2\sin x \cos x]\, dx$

$= x - 2\int \sin x \cos x\, dx$

(Let $u = \sin x \xrightarrow{D} du = \cos x\, dx$)

$= x - 2\int u\, du$

$= x - 2 \cdot \frac{1}{2} u^2 + C$

$= x - (\sin x)^2 + C$

8.) $\int (\cos x + \sin x)(\sin x - \cos x)^3\, dx$

(Let $u = \sin x - \cos x \xrightarrow{D}$
$du = (\cos x - (-\sin x))\, dx = (\cos x + \sin x)\, dx$)

$= \int u^3\, du = \frac{1}{4} u^4 + C$

$= \frac{1}{4}(\sin x - \cos x)^4 + C$

9.) $\int (\sec x + \tan x)^2\, dx$

$= \int (\sec^2 x + 2\sec x \tan x + \tan^2 x)\, dx$

(Use Trig Identity 2.)

$$= \int [\sec^2 x + 2\sec x \tan x + (\sec^2 x - 1)]\, dx$$

$$= \int [2\sec^2 x - 1 + 2\sec x \tan x]\, dx$$

$$= 2\tan x - x + 2\sec x + C$$

(*) 10.) $\int \sec x (\sec x + \tan x)^5 \, dx$

(Let $u = \sec x + \tan x \longrightarrow$

$du = (\sec x \tan x + \sec^2 x)\, dx$

$= \sec x (\tan x + \sec x)\, dx \longrightarrow$

$\frac{1}{\sec x + \tan x}\, du = \sec x\, dx \longrightarrow$

$\frac{1}{u}\, du = \sec x\, dx$)

$$= \int u^5 \cdot \frac{1}{u}\, du = \int u^4\, du$$

$$= \frac{1}{5}u^5 + C = \frac{1}{5}(\sec x + \tan x)^5 + C$$

11.) $\int \frac{1 + \sin x}{\cos x}\, dx = \int \left[\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right] dx$

$$= \int (\sec x + \tan x)\, dx$$

$= \ln|\sec x + \tan x| + \ln|\sec x| + C$

12.) $\int \sin^3 x \, dx = \int \sin x \cdot \sin^2 x \, dx$

(Use Trig Identity 1.)

$= \int \sin x (1 - \cos^2 x) \, dx$

$= \int (\sin x - \sin x \cdot \cos^2 x) \, dx$

$= -\cos x - \frac{-1}{3} \cos^3 x + C$

13.) $\int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx$

(Use Trig Identity 2.)

$= \int \sec^2 x (1 + \tan^2 x) \, dx$

$= \int (\sec^2 x + \sec^2 x \tan^2 x) \, dx$

$= \tan x + \frac{1}{3} \tan^3 x + C$

14.) $\int \sin^2 x \, dx$

(Use Trig Identity 5.: $\cos 2x = 1 - 2\sin^2 x$

$\rightarrow 2\sin^2 x = 1 - \cos 2x \rightarrow \sin^2 x = \frac{1}{2}(1 - \cos 2x)$)

$= \frac{1}{2}\int (1-\cos 2x)\,dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

14.) $\int \tan^3 x\,dx = \int \tan x \cdot \tan^2 x\,dx$

(Use Trig Identity 2.)

$= \int \tan x\,(\sec^2 x - 1)\,dx$

$= \int (\tan x \cdot \sec^2 x - \tan x)\,dx$

$= \frac{1}{2}\tan^2 x - \ln|\sec x| + C$

15.) $\int \sec^5 x \cdot \tan x\,dx$

$= \int \sec^4 x \cdot \sec x \tan x\,dx$

$= \frac{1}{5}\sec^5 x + C$