Math 16B
Kouba
Continuous Probability, Definitions

Let $f(x)$ be a probability density function for a continuous random variable, x, over the interval $a \le x \le b$.

1.) The <u>MEAN</u>, μ, or <u>EXPECTED VALUE</u>, $E(x)$, of x is

$$\mu = E(x) = \int_a^b x f(x)\, dx \quad .$$

2.) The <u>MEDIAN</u>, m, of x is the value m in the interval $a \le x \le b$ which satisfies

$$P(a \le x \le m) = 0.5 = 1/2$$

or

$$\int_a^m f(x)\, dx = 0.5 = 1/2 \quad .$$

3.) The <u>VARIANCE</u>, $V(x)$, of x is

$$V(x) = \int_a^b (x-\mu)^2 f(x)\, dx = \int_a^b x^2 f(x)\, dx - \mu^2 \, .$$

4.) The <u>STANDARD DEVIATION</u>, σ, of x is

$$\sigma = \sqrt{V(x)} \quad .$$

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P.D.F. Example

<u>Example</u>: assume that the number of hours x that a UC Davis student spends on Twitter each day is given by the probability density function

$$f(x) = \frac{1/2}{x} \text{ for } 1 \le x \le e^2 \approx 7.39$$

1.) Verify that f is a p.d.f.

2.) Find the mean, $\mu = E(x)$.

3.) Find the median, m.

4.) Find the variance, $V(x)$.

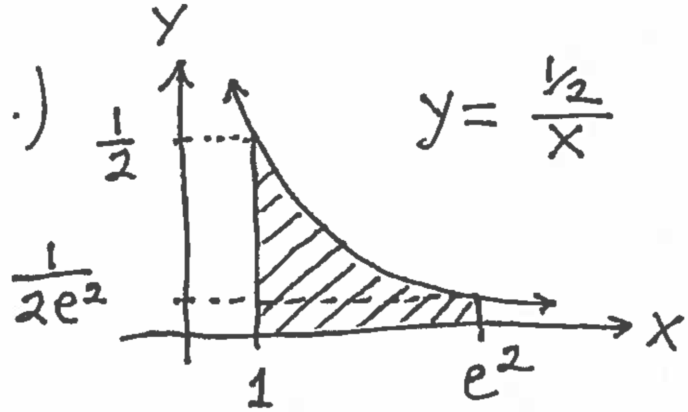
5.) Find the standard deiviation, σ.

6.) Find $P(1 \le X \le 4)$.

7.) Find $P(5 \le X \le 7)$.

8.) Find $P(X = 3)$.

1.)



i.) Clearly, $f(x) \geq 0$ for $1 \leq x \leq e^2$.

ii.) $\int_1^{e^2} \frac{1/2}{x}\,dx = \frac{1}{2}\ln|x|\Big|_1^{e^2}$

$= \frac{1}{2}\ln e^2 - \frac{1}{2}\ln 1$

$= \frac{1}{2}(2) - \frac{1}{2}(0) = 1$

2.) $\mu = E(x) = \int_1^{e^2} x \cdot \frac{1/2}{x}\,dx$

$= \int_1^{e^2} \frac{1}{2}\,dx = \frac{1}{2}x\Big|_1^{e^2}$

$= \frac{1}{2}e^2 - \frac{1}{2} \approx 3.19$ hrs.

3.) $\int_1^{m} \frac{1/2}{x}\,dx = \frac{1}{2}\ln|x|\Big|_1^{m}$

$= \frac{1}{2}\ln m - \frac{1}{2}\overset{0}{\cancel{\ln 1}} = \frac{1}{2} \rightarrow$

$\ln m = 1 \rightarrow m = e \approx 2.718$ hrs.

4.) $V(x) = \int_1^{e^2} x^2 \cdot \frac{1/2}{x}\,dx - \mu^2$

$= \int_1^{e^2} \frac{1}{2}x\,dx - e^2$

$= \frac{1}{2}\cdot\frac{1}{2}x^2 \Big|_1^{e^2} - e^2$

$= \frac{1}{4}(e^2)^2 - \frac{1}{4}(1)^2 - e^2$

$= \frac{1}{4}e^4 - \frac{1}{4} - e^2 \approx 6.01$ hrs.^{2}

5.) $\sigma = \sqrt{V(x)} = \sqrt{6.01} \approx 2.45$ hrs.

6.) $P(1 \le x \le 4) = \int_1^4 \frac{1/2}{x}\,dx$

$= \frac{1}{2}\ln|x| \Big|_1^4 = \frac{1}{2}\ln 4 - \frac{1}{2}\cancel{\ln 1}^{\,0}$

$= \frac{1}{2}\ln 4 \approx 0.693 = 69.3\%$

7.) $P(5 \le x \le 7) = \int_5^7 \frac{1/2}{x}\,dx$

$= \frac{1}{2}\ln|x| \Big|_5^7 = \frac{1}{2}\ln 7 - \frac{1}{2}\ln 5$

$\approx 0.168 = 16.8\%$

8.) $P(x = 3) = P(3 \le x \le 3) = \int_3^3 \frac{1/2}{x}\,dx = 0$