PREREQUISITE REVIEW 5.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, rewrite the expression using rational exponents.

1. $\dfrac{\sqrt{x}}{x}$
2. $\sqrt[3]{2x}(2x)$
3. $\sqrt{5x^3} + \sqrt{x^5}$
4. $\dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$
5. $\dfrac{(x+1)^3}{\sqrt{x+1}}$
6. $\dfrac{\sqrt{x}}{\sqrt[3]{x}}$

In Exercises 7–10, let $(x, y) = (2, 2)$, and solve the equation for C.

7. $y = x^2 + 5x + C$
8. $y = 3x^3 - 6x + C$
9. $y = -16x^2 + 26x + C$
10. $y = -\frac{1}{4}x^4 - 2x^2 + C$

EXERCISES 5.1

In Exercises 1–8, verify the statement by showing that the derivative of the right side is equal to the integrand of the left side.

1. $\displaystyle\int \left(-\frac{9}{x^4}\right) dx = \frac{3}{x^3} + C$
2. $\displaystyle\int \frac{4}{\sqrt{x}}\, dx = 8\sqrt{x} + C$
3. $\displaystyle\int \left(4x^3 - \frac{1}{x^2}\right) dx = x^4 + \frac{1}{x} + C$
4. $\displaystyle\int \left(1 - \frac{1}{\sqrt[3]{x^2}}\right) dx = x - 3\sqrt[3]{x} + C$
5. $\displaystyle\int 2\sqrt{x}(x-3)\, dx = \frac{4x^{3/2}(x-5)}{5} + C$
6. $\displaystyle\int 4\sqrt{x}(x^2-2)\, dx = \frac{8x^{3/2}(3x^2-14)}{21} + C$
7. $\displaystyle\int \frac{x^2-1}{x^{3/2}}\, dx = \frac{2(x^2+3)}{3\sqrt{x}} + C$
8. $\displaystyle\int \frac{2x-1}{x^{4/3}}\, dx = \frac{3(x+1)}{\sqrt[3]{x}} + C$

In Exercises 9–20, find the indefinite integral and check your result by differentiation.

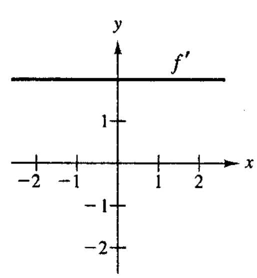
9. $\int 6\, dx$
10. $\int -4\, dx$
11. $\int 5t^2\, dt$
12. $\int 3t^4\, dt$
13. $\int 5x^{-3}\, dx$
14. $\int 4y^{-3}\, dy$
15. $\int du$
16. $\int dr$
17. $\int e\, dt$
18. $\int e^3\, dy$
19. $\int y^{3/2}\, dy$
20. $\int v^{-1/2}\, dv$

In Exercises 21–28, find the indefinite integral using the columns in Example 4 as a model. Use a symbolic integration utility to verify your results.

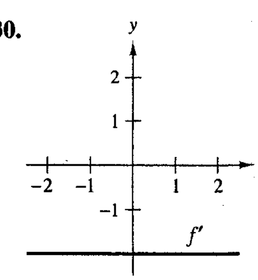
21. $\int \sqrt[3]{x}\, dx$
22. $\displaystyle\int \frac{1}{x^2}\, dx$
23. $\displaystyle\int \frac{1}{x\sqrt{x}}\, dx$
24. $\displaystyle\int \frac{1}{x^2\sqrt{x}}\, dx$
25. $\int x(x^2+3)\, dx$
26. $\int t(t^2+2)\, dt$
27. $\displaystyle\int \frac{1}{2x^3}\, dx$
28. $\displaystyle\int \frac{1}{8x^3}\, dx$

In Exercises 29–32, find two functions that have the given derivative, and sketch the graph of each. (There is more than one correct answer.)

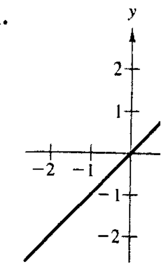
29.



30.



31.



In Exercises 33–42,
result by differentiat

33. $\int (x^3+2)\, dx$
35. $\int \left(\sqrt[3]{x} - \frac{1}{2\sqrt[3]{x}}\right.$
37. $\int \left(\sqrt[3]{x^2}+1\right) d$
39. $\displaystyle\int \frac{1}{3x^4}\, dx$
41. $\displaystyle\int \frac{2x^3+1}{x^3}\, dx$

In Exercises 43–48, u
indefinite integral.

43. $\int u(3u^2+1)\, du$
45. $\int (x-1)(6x -$
47. $\int y^2\sqrt{y}\, dy$

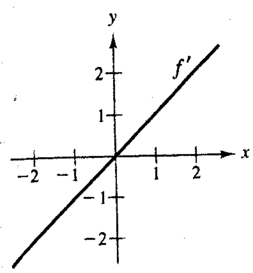
In Exercises 49–54, f
satisfies the differenti

49. $f'(x) = 3\sqrt{x} +$
50. $f'(x) = \frac{1}{5}x - 2;$
51. $f'(x) = 6x(x-1$
52. $f'(x) = (2x-3)$
53. $f'(x) = \dfrac{2-x}{x^3},\ x$
54. $f'(x) = \dfrac{x^2-5}{x^2},$

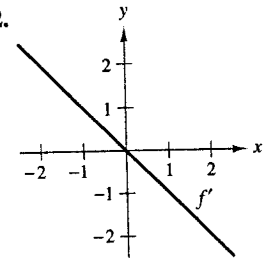
In Exercises 55 and 56,
which is a general solu
the equation of the pa
indicated point.

55. $\dfrac{dy}{dx} = -5x - 2$

31.



32.



In Exercises 33–42, find the indefinite integral and check your result by differentiation.

33. $\int (x^3 + 2)\, dx$

34. $\int (x^2 - 2x + 3)\, dx$

35. $\int \left(\sqrt[3]{x} - \frac{1}{2\sqrt[3]{x}}\right) dx$

36. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

37. $\int \left(\sqrt[3]{x^2} + 1\right) dx$

38. $\int (\sqrt[4]{x^3} + 1)\, dx$

39. $\int \frac{1}{3x^4}\, dx$

40. $\int \frac{1}{4x^2}\, dx$

41. $\int \frac{2x^3 + 1}{x^3}\, dx$

42. $\int \frac{t^2 + 2}{t^2}\, dt$

In Exercises 43–48, use a symbolic integration utility to find the indefinite integral.

43. $\int u(3u^2 + 1)\, du$

44. $\int \sqrt{x}(x + 1)\, dx$

45. $\int (x - 1)(6x - 5)\, dx$

46. $\int (2t^2 - 1)^2\, dt$

47. $\int y^2\sqrt{y}\, dy$

48. $\int (1 + 3t)t^2\, dt$

In Exercises 49–54, find the particular solution $y = f(x)$ that satisfies the differential equation and initial condition.

49. $f'(x) = 3\sqrt{x} + 3;\quad f(1) = 4$

50. $f'(x) = \frac{1}{5}x - 2;\quad f(10) = -10$

51. $f'(x) = 6x(x - 1);\quad f(1) = -1$

52. $f'(x) = (2x - 3)(2x + 3);\quad f(3) = 0$

53. $f'(x) = \frac{2 - x}{x^3},\ x > 0;\quad f(2) = \frac{3}{4}$

54. $f'(x) = \frac{x^2 - 5}{x^2},\ x > 0;\quad f(1) = 2$

In Exercises 55 and 56, you are shown a family of graphs, each of which is a general solution of the given differential equation. Find the equation of the particular solution that passes through the indicated point.

55. $\frac{dy}{dx} = -5x - 2$

56. $\frac{dy}{dx} = 2(x - 1)$

Figure for 55

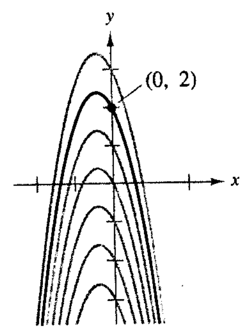
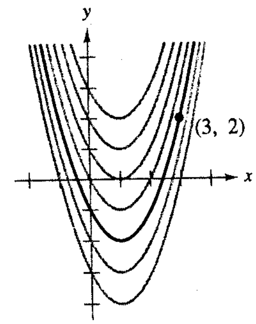


Figure for 56



In Exercises 57 and 58, find the equation of the function f whose graph passes through the point.

	Derivative	*Point*
57.	$f'(x) = 6\sqrt{x} - 10$	(4, 2)
58.	$f'(x) = \frac{6}{x^2}$	(2, 5)

In Exercises 59–62, find a function f that satisfies the conditions.

59. $f''(x) = 2,\quad f'(2) = 5,\quad f(2) = 10$

60. $f''(x) = x^2,\quad f'(0) = 6,\quad f(0) = 3$

61. $f''(x) = x^{-2/3},\quad f'(8) = 6,\quad f(0) = 0$

62. $f''(x) = x^{-3/2},\quad f'(1) = 2,\quad f(9) = -4$

Cost In Exercises 63–66, find the cost function for the marginal cost and fixed cost.

	Marginal Cost	*Fixed Cost (x = 0)*
63.	$\frac{dC}{dx} = 85$	\$5500
64.	$\frac{dC}{dx} = \frac{1}{50}x + 10$	\$1000
65.	$\frac{dC}{dx} = \frac{1}{20\sqrt{x}} + 4$	\$750
66.	$\frac{dC}{dx} = \frac{\sqrt[4]{x}}{10} + 10$	\$2300

Demand Function In Exercises 67–70, find the revenue and demand functions for the given marginal revenue. (Use the fact that $R = 0$ when $x = 0$.)

67. $\frac{dR}{dx} = 225 - 3x$

68. $\frac{dR}{dx} = 310 - 4x$

69. $\frac{dR}{dx} = 225 + 2x - x^2$

70. $\frac{dR}{dx} = 100 - 6x - 2x^2$

Profit In Exercises 71–74, find the profit function for the given marginal profit and initial condition.

Marginal Profit	*Initial Condition*
71. $\frac{dP}{dx} = -18x + 1650$	$P(15) = \$22{,}725$
72. $\frac{dP}{dx} = -40x + 250$	$P(5) = \$650$
73. $\frac{dP}{dx} = -24x + 805$	$P(12) = \$8000$
74. $\frac{dP}{dx} = -30x + 920$	$P(8) = \$6500$

Vertical Motion In Exercises 75–78, use $a(t) = -32$ feet per second per second as the acceleration due to gravity.

75. A ball is thrown vertically upward with an initial velocity of 60 feet per second. How high will the ball go?

76. The Grand Canyon is 6000 feet deep at the deepest part. A rock is dropped from this height. Express the height of the rock as a function of the time t (in seconds). How long will it take the rock to hit the canyon floor?

77. With what initial velocity must an object be thrown upward from the ground to reach the height of the Washington Monument (550 feet)?

78. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at an instant when the balloon is 64 feet above the ground.

(a) How many seconds after its release will the bag strike the ground?

(b) With what velocity will the bag strike the ground?

79. ***Cost*** A company produces a product for which the marginal cost of producing x units is modeled by

$$\frac{dC}{dx} = 2x - 12$$

and the fixed costs are \$125.

(a) Find the total cost function and the average cost function.

(b) Find the total cost of producing 50 units.

(c) In part (b), how much of the total cost is fixed? How much is variable? Give examples of fixed costs associated with the manufacturing of a product. Give examples of variable costs.

80. ***Population Growth*** The growth rate of Horry County in South Carolina can be modeled by

$$\frac{dP}{dt} = 105.74t + 2639.3$$

where t is the time in years, with $t = 0$ corresponding to 1970. The county's population was 196,629 in 2000. *(Source: U.S. Census Bureau)*

(a) Find the model for Horry County's population.

(b) Use the model to predict the population in 2010. Does your answer seem reasonable? Explain your reasoning.

81. ***Vital Statistics*** The rate of increase of the number of married couples M (in thousands) in the United States from 1970 to 2000 can be modeled by

$$\frac{dM}{dt} = 0.636t^2 - 28.48t + 632.7$$

where t is the time in years, with $t = 0$ corresponding to 1970. The number of married couples in 2000 was 56,497 thousand. *(Source: U.S. Census Bureau)*

(a) Find the model for the number of married couples in the United States.

(b) Use the model to predict the number of married couples in the United States in 2010. Does your answer seem reasonable? Explain your reasoning.

82. ***Economics: Marginal Benefits and Costs*** The table gives the marginal benefit and marginal cost of producing x products for a given company. Plot the points in each column and use the *regression* feature of a graphing utility to find a linear model for marginal benefit and a quadratic model for marginal cost. Then use integration to find the benefit B and cost C equations. Assume $B(0) = 0$ and $C(0) = 425$. Finally, find the intervals in which the benefit exceeds the cost of producing x products, and make a recommendation for how many products the company should produce based on your findings. *(Source: Adapted from Taylor, Economics, Fourth Edition)*

Number of products	1	2	3	4	5
Marginal benefit	330	320	290	270	250
Marginal cost	150	120	100	110	120

Number of products	6	7	8	9	10
Marginal benefit	230	210	190	170	160
Marginal cost	140	160	190	250	320

83. ***Research Project*** Use your school's library, the Internet, or some other reference source to research a company that markets a natural resource. Find data on the revenue of the company and on the consumption of the resource. Then find a model for each. Is the company's revenue related to the consumption of the resource? Explain your reasoning.

PREREQUISITE REVIEW 5.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–10, find the indefinite integral.

1. $\int (2x^3 + 1)\,dx$
2. $\int (x^{1/2} + 3x - 4)\,dx$
3. $\int \frac{1}{x^2}\,dx$
4. $\int \frac{1}{3t^3}\,dt$
5. $\int (1 + 2t)t^{3/2}\,dt$
6. $\int \sqrt{x}(2x - 1)\,dx$
7. $\int \frac{5x^3 + 2}{x^2}\,dx$
8. $\int \frac{2x^2 - 5}{x^4}\,dx$
9. $\int (x^2 + 1)^2\,dx$
10. $\int (x^3 - 2x + 1)^2\,dx$

In Exercises 11–14, simplify the expression.

11. $\left(-\frac{5}{4}\right)\frac{(x - 2)^4}{4}$
12. $\left(\frac{1}{6}\right)\frac{(x - 1)^{-2}}{-2}$
13. $(6)\frac{(x^2 + 3)^{2/3}}{2/3}$
14. $\left(\frac{5}{2}\right)\frac{(1 - x^3)^{-1/2}}{-1/2}$

EXERCISES 5.2

In Exercises 1–8, identify u and du/dx for the integral $\int u^n(du/dx)\,dx$.

1. $\int (5x^2 + 1)^2(10x)\,dx$
2. $\int (3 - 4x^2)^3(-8x)\,dx$
3. $\int \sqrt{1 - x^2}(-2x)\,dx$
4. $\int 3x^2\sqrt{x^3 + 1}\,dx$
5. $\int \left(4 + \frac{1}{x^2}\right)^5\left(\frac{-2}{x^3}\right)dx$
6. $\int \frac{1}{(1 + 2x)^2}(2)\,dx$
7. $\int (1 + \sqrt{x})^3\left(\frac{1}{2\sqrt{x}}\right)dx$
8. $\int (4 - \sqrt{x})^2\left(\frac{-1}{2\sqrt{x}}\right)dx$

In Exercises 9–28, find the indefinite integral and check the result by differentiation.

9. $\int (1 + 2x)^4(2)\,dx$
10. $\int (x^2 - 1)^3(2x)\,dx$
11. $\int \sqrt{5x^2 - 4}(10x)\,dx$
12. $\int \sqrt{3 - x^3}(3x^2)\,dx$
13. $\int (x - 1)^4\,dx$
14. $\int (x - 3)^{5/2}\,dx$
15. $\int x(x^2 - 1)^7\,dx$
16. $\int x(1 - 2x^2)^3\,dx$
17. $\int \frac{x^2}{(1 + x^3)^2}\,dx$
18. $\int \frac{x^2}{(x^3 - 1)^2}\,dx$
19. $\int \frac{x + 1}{(x^2 + 2x - 3)^2}\,dx$
20. $\int \frac{6x}{(1 + x^2)^3}\,dx$
21. $\int \frac{x - 2}{\sqrt{x^2 - 4x + 3}}\,dx$
22. $\int \frac{4x + 6}{(x^2 + 3x + 7)^3}\,dx$
23. $\int 5u\sqrt[3]{1 - u^2}\,du$
24. $\int u^3\sqrt{u^4 + 2}\,du$
25. $\int \frac{4y}{\sqrt{1 + y^2}}\,dy$
26. $\int \frac{x^2}{\sqrt{1 - x^3}}\,dx$
27. $\int \frac{-3}{\sqrt{2t + 3}}\,dt$
28. $\int \frac{t + 2t^2}{\sqrt{t}}\,dt$

In Exercises 29–34, use a symbolic integration utility to find the indefinite integral.

29. $\int \frac{x^3}{\sqrt{1 - x^4}}\,dx$
30. $\int \frac{3x}{\sqrt{1 - 4x^2}}\,dx$
31. $\int \left(1 + \frac{4}{t^2}\right)^2\left(\frac{1}{t^3}\right)dt$
32. $\int \left(1 + \frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt$
33. $\int (x^3 + 3x)(x^2 + 1)\,dx$
34. $\int (3 - 2x - 4x^2)(1 + 4x)\,dx$

In Exercises 35–42, use formal substitution (as illustrated in Example 5) to find the indefinite integral.

35. $\int x(6x^2 - 1)^3\,dx$ **36.** $\int x^2(1 - x^3)^2\,dx$

37. $\int x^2(2 - 3x^3)^{3/2}\,dx$ **38.** $\int t\sqrt{t^2 + 1}\,dt$

39. $\int \frac{x}{\sqrt{x^2 + 25}}\,dx$ **40.** $\int \frac{3}{\sqrt{2x + 1}}\,dx$

41. $\int \frac{x^2 + 1}{\sqrt{x^3 + 3x + 4}}\,dx$ **42.** $\int \sqrt{x}(4 - x^{3/2})^2\,dx$

In Exercises 43–46, (a) perform the integration in two ways: once using the Simple Power Rule and once using the General Power Rule. (b) Explain the difference in the results. (c) Which method do you prefer? Explain your reasoning.

43. $\int (2x - 1)^2\,dx$ **44.** $\int (3 - 2x)^2\,dx$

45. $\int x(x^2 - 1)^2\,dx$ **46.** $\int x(2x^2 + 1)^2\,dx$

47. Find the equation of the function f whose graph passes through the point $\left(0, \frac{4}{3}\right)$ and whose derivative is

$$f'(x) = x\sqrt{1 - x^2}.$$

48. Find the equation of the function f whose graph passes through the point $\left(0, \frac{7}{3}\right)$ and whose derivative is

$$f'(x) = x\sqrt{1 - x^2}.$$

49. ***Cost*** The marginal cost of a product is modeled by

$$\frac{dC}{dx} = \frac{4}{\sqrt{x + 1}}.$$

When $x = 15$, $C = 50$.

(a) Find the cost function.

(b) Use a graphing utility to graph dC/dx and C in the same viewing window.

50. ***Cost*** The marginal cost of a product is modeled by

$$\frac{dC}{dx} = \frac{12}{\sqrt[3]{12x + 1}}.$$

When $x = 13$, $C = 100$.

(a) Find the cost function.

(b) Use a graphing utility to graph dC/dx and C in the same viewing window.

Supply Function In Exercises 51 and 52, find the supply function $x = f(p)$ that satisfies the initial conditions.

51. $\frac{dx}{dp} = p\sqrt{p^2 - 25}$, $x = 600$ when $p = \$13$

52. $\frac{dx}{dp} = \frac{10}{\sqrt{p - 3}}$, $x = 100$ when $p = \$3$

Demand Function In Exercises 53 and 54, find the demand function $x = f(p)$ that satisfies the initial conditions.

53. $\frac{dx}{dp} = -\frac{6000p}{(p^2 - 16)^{3/2}}$, $x = 5000$ when $p = \$5$

54. $\frac{dx}{dp} = -\frac{400}{(0.02p - 1)^3}$, $x = 10{,}000$ when $p = \$100$

55. ***Gardening*** An evergreen nursery usually sells a type of shrub after 5 years of growth and shaping. The growth rate during those 5 years is approximated by

$$\frac{dh}{dt} = \frac{17.6t}{\sqrt{17.6t^2 + 1}}$$

where t is time in years and h is height in inches. The seedlings are 6 inches tall when planted ($t = 0$).

(a) Find the height function.

(b) How tall are the shrubs when they are sold?

56. ***Cash Flow*** The rate of disbursement dQ/dt of a \$2 million federal grant is proportional to the square of $100 - t$, where t is the time (in days, $0 \le t \le 100$) and Q is the amount that remains to be disbursed. Find the amount that remains to be disbursed after 50 days. Assume that the entire grant will be disbursed after 100 days.

Marginal Propensity to Consume In Exercises 57 and 58, (a) use the marginal propensity to consume, dQ/dx, to write Q as a function of x, where x is the income (in dollars) and Q is the income consumed (in dollars). Assume that 100% of the income is consumed for families that have annual incomes of \$20,000 or less. (b) Use the result of part (a) to complete the table showing the income consumed and the income saved, $x - Q$, for various incomes. (c) Use a graphing utility to represent graphically the income consumed and saved.

x	20,000	50,000	100,000	150,000
Q				
$x - Q$				

57. $\frac{dQ}{dx} = \frac{0.95}{(x - 19{,}999)^{0.05}}$, $x \ge 20{,}000$

58. $\frac{dQ}{dx} = \frac{0.93}{(x - 19{,}999)^{0.07}}$, $x \ge 20{,}000$

In Exercises 59 and 60, use a symbolic integration utility to find the indefinite integral. Verify the result by differentiating.

59. $\int \frac{1}{\sqrt{x} + \sqrt{x + 1}}\,dx$ **60.** $\int \frac{x}{\sqrt{3x + 2}}\,dx$

5.

Using the Ex

Each of the differ
integration rule.

Integrals of E

Let u be a diff

$\int e^x\,d$

$\int e^u \frac{du}{dx}\,d$

EXAMPLE 1

Find each indefini

(a) $\int 2e^x\,dx$

(b) $\int 2e^{2x}\,dx$

(c) $\int (e^x + x)\,dx$

SOLUTION

(a) $\int 2e^x\,dx = 2$

$= 2e$

(b) $\int 2e^{2x}\,dx = \int$

$= \int$

$= e^2$

(c) $\int (e^x + x)\,dx$

You can check each