PREREQUISITE REVIEW 5.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, find the domain of the function.

1. $y = \ln(2x - 5)$

2. $y = \ln(x^2 - 5x + 6)$

In Exercises 3–6, use long division to rewrite the quotient.

3. $\dfrac{x^2 + 4x + 2}{x + 2}$

4. $\dfrac{x^2 - 6x + 9}{x - 4}$

5. $\dfrac{x^3 + 4x^2 - 30x - 4}{x^2 - 4x}$

6. $\dfrac{x^4 - x^3 + x^2 + 15x + 2}{x^2 + 5}$

In Exercises 7–10, evaluate the integral.

7. $\displaystyle\int \left(x^3 + \frac{1}{x^2}\right) dx$

8. $\displaystyle\int \frac{x^2 + 2x}{x}\, dx$

9. $\displaystyle\int \frac{x^3 + 4}{x^2}\, dx$

10. $\displaystyle\int \frac{x + 3}{x^3}\, dx$

EXERCISES 5.3

n Exercises 1–12, use the Exponential Rule to find the indefinite ntegral.

1. $\displaystyle\int 2e^{2x}\, dx$

2. $\displaystyle\int -3e^{-3x}\, dx$

3. $\displaystyle\int e^{4x}\, dx$

4. $\displaystyle\int e^{-0.25x}\, dx$

5. $\displaystyle\int 9xe^{-x^2}\, dx$

6. $\displaystyle\int 3xe^{0.5x^2}\, dx$

7. $\displaystyle\int 5x^2 e^{x^3}\, dx$

8. $\displaystyle\int (2x + 1)e^{x^2 + x}\, dx$

9. $\displaystyle\int (x^2 + 2x)e^{x^3 + 3x^2 - 1}\, dx$

10. $\displaystyle\int 3(x - 4)e^{x^2 - 8x}\, dx$

11. $\displaystyle\int 5e^{2-x}\, dx$

12. $\displaystyle\int 3e^{-(x+1)}\, dx$

n Exercises 13–28, use the Log Rule to find the indefinite integral.

13. $\displaystyle\int \frac{1}{x + 1}\, dx$

14. $\displaystyle\int \frac{1}{x - 5}\, dx$

15. $\displaystyle\int \frac{1}{3 - 2x}\, dx$

16. $\displaystyle\int \frac{1}{6x - 5}\, dx$

17. $\displaystyle\int \frac{2}{3x + 5}\, dx$

18. $\displaystyle\int \frac{5}{2x - 1}\, dx$

19. $\displaystyle\int \frac{x}{x^2 + 1}\, dx$

20. $\displaystyle\int \frac{x^2}{3 - x^3}\, dx$

21. $\displaystyle\int \frac{x^2}{x^3 + 1}\, dx$

22. $\displaystyle\int \frac{x}{x^2 + 4}\, dx$

23. $\displaystyle\int \frac{x + 3}{x^2 + 6x + 7}\, dx$

24. $\displaystyle\int \frac{x^2 + 2x + 3}{x^3 + 3x^2 + 9x + 1}\, dx$

25. $\displaystyle\int \frac{1}{x \ln x}\, dx$

26. $\displaystyle\int \frac{1}{x(\ln x)^2}\, dx$

27. $\displaystyle\int \frac{e^{-x}}{1 - e^{-x}}\, dx$

28. $\displaystyle\int \frac{e^x}{1 + e^x}\, dx$

In Exercises 29–38, use a symbolic integration utility to find the indefinite integral.

29. $\displaystyle\int \frac{1}{x^2} e^{2/x}\, dx$

30. $\displaystyle\int \frac{1}{x^3} e^{1/4x^2}\, dx$

31. $\displaystyle\int \frac{1}{\sqrt{x}} e^{\sqrt{x}}\, dx$

32. $\displaystyle\int \frac{e^{1/\sqrt{x}}}{x^{3/2}}\, dx$

33. $\displaystyle\int (e^x - 2)^2\, dx$

34. $\displaystyle\int (e^x - e^{-x})^2\, dx$

35. $\displaystyle\int \frac{e^{-x}}{1 + e^{-x}}\, dx$

36. $\displaystyle\int \frac{3e^x}{2 + e^x}\, dx$

37. $\displaystyle\int \frac{4e^{2x}}{5 - e^{2x}}\, dx$

38. $\displaystyle\int \frac{-e^{3x}}{2 - e^{3x}}\, dx$

In Exercises 39–54, use any
to find the indefinite integr

39. $\displaystyle\int \frac{e^{2x} + 2e^x + 1}{e^x}\, dx$

41. $\displaystyle\int e^x\sqrt{1 - e^x}\, dx$

43. $\displaystyle\int \frac{1}{(x - 1)^2}\, dx$

45. $\displaystyle\int 4e^{2x-1}\, dx$

47. $\displaystyle\int \frac{x^3 - 8x}{2x^2}\, dx$

49. $\displaystyle\int \frac{2}{1 + e^{-x}}\, dx$

51. $\displaystyle\int \frac{x^2 + 2x + 5}{x - 1}\, dx$

53. $\displaystyle\int \frac{1 + e^{-x}}{1 + xe^{-x}}\, dx$

In Exercises 55 and 56, find
graph passes through the p

55. $f'(x) = \dfrac{x^2 + 4x + 3}{x - 1}$;

56. $f'(x) = \dfrac{x^3 - 4x^2 + 3}{x - 3}$

57. ***Biology*** A populatio

$$\frac{dP}{dt} = \frac{3000}{1 + 0.25t}$$

where t is the time in
1000.

(a) Write an equatio
terms of the time

(b) What is the popul

(c) After how many d

58. ***Biology*** Because o
trout population in a l
change can be modele

$$\frac{dP}{dt} = -125e^{-t/20}$$

where t is the time in
2500.

(a) Write an equatio
terms of the time

(b) What is the popul

(c) According to this
entire trout popul

In Exercises 39–54, use any basic integration formula or formulas to find the indefinite integral.

39. $\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx$

40. $\int (6x + e^x)\sqrt{3x^2 + e^x}\,dx$

41. $\int e^x\sqrt{1 - e^x}\,dx$

42. $\int \frac{2(e^x - e^{-x})}{(e^x + e^{-x})^2}\,dx$

43. $\int \frac{1}{(x-1)^2}\,dx$

44. $\int \frac{1}{\sqrt{x+1}}\,dx$

45. $\int 4e^{2x-1}\,dx$

46. $\int (5e^{-2x} + 1)\,dx$

47. $\int \frac{x^3 - 8x}{2x^2}\,dx$

48. $\int \frac{x-1}{4x}\,dx$

49. $\int \frac{2}{1 + e^{-x}}\,dx$

50. $\int \frac{3}{1 + e^{-3x}}\,dx$

51. $\int \frac{x^2 + 2x + 5}{x - 1}\,dx$

52. $\int \frac{x-3}{x+3}\,dx$

53. $\int \frac{1 + e^{-x}}{1 + xe^{-x}}\,dx$

54. $\int \frac{5}{e^{-5x} + 7}\,dx$

In Exercises 55 and 56, find the equation of the function f whose graph passes through the point.

55. $f'(x) = \frac{x^2 + 4x + 3}{x - 1}$; $(2, 4)$

56. $f'(x) = \frac{x^3 - 4x^2 + 3}{x - 3}$; $(4, -1)$

57. ***Biology*** A population of bacteria is growing at the rate of

$$\frac{dP}{dt} = \frac{3000}{1 + 0.25t}$$

where t is the time in days. When $t = 0$, the population is 1000.

(a) Write an equation that models the population P in terms of the time t.

(b) What is the population after 3 days?

(c) After how many days will the population be 12,000?

58. ***Biology*** Because of an insufficient oxygen supply, the trout population in a lake is dying. The population's rate of change can be modeled by

$$\frac{dP}{dt} = -125e^{-t/20}$$

where t is the time in days. When $t = 0$, the population is 2500.

(a) Write an equation that models the population P in terms of the time t.

(b) What is the population after 15 days?

(c) According to this model, how long will it take for the entire trout population to die?

59. ***Demand*** The marginal price for the demand of a product can be modeled by $dp/dx = 0.1e^{-x/500}$, where x is the quantity demanded. When the demand is 600 units, the price is \$30.

(a) Find the demand function, $p = f(x)$.

(b) Use a graphing utility to graph the demand function. Does price increase or decrease as demand increases?

(c) Use the *zoom* and *trace* features of the graphing utility to find the quantity demanded when the price is \$22.

60. ***Revenue*** The marginal revenue for the sale of a product can be modeled by

$$\frac{dR}{dx} = 50 - 0.02x + \frac{100}{x+1}$$

where x is the quantity demanded.

(a) Find the revenue function.

(b) Use a graphing utility to graph the revenue function.

(c) Find the revenue when 1500 units are sold.

(d) Use the *zoom* and *trace* features of the graphing utility to find the number of units sold when the revenue is \$60,230.

61. ***Average Salary*** From 1995 through 2002, the average salary for superintendents S (in dollars) in the United States changed at the rate of

$$\frac{dS}{dt} = 2621.7e^{0.07t}$$

where $t = 5$ corresponds to 1995. In 2001, the average salary for superintendents was \$118,496. *(Source: Educational Research Service)*

(a) Write a model that gives the average salary for superintendents per year.

(b) Use the model to find the average salary for superintendents in 1999.

62. ***Sales*** The rate of change in sales for The Yankee Candle Company from 1998 through 2003 can be modeled by

$$\frac{dS}{dt} = 1.04t + \frac{544.694}{t}$$

where S is the sales (in millions) and $t = 8$ corresponds to 1998. In 1999, the sales for The Yankee Candle Company were \$256.6 million. *(Source: The Yankee Candle Company)*

(a) Find a model for sales from 1998 through 2003.

(b) Find The Yankee Candle Company's sales in 2002.

True or False? In Exercises 63 and 64, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

63. $(\ln x)^{1/2} = \frac{1}{2}(\ln x)$

64. $\int \ln x = \left(\frac{1}{x}\right) + C$

rvals over a period of
payroll savings plans,
'ement accounts. The
he interest earned and

per year (where
ıpounded continu-
, then the

ıity

al retirement account
ır IRA after 15 years?
$c(t) = 2000$. So, the

PREREQUISITE REVIEW 5.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the indefinite integral.

1. $\int (3x + 7)\,dx$

2. $\int (x^{3/2} + 2\sqrt{x})\,dx$

3. $\int \frac{1}{5x}\,dx$

4. $\int e^{-6x}\,dx$

In Exercises 5 and 6, evaluate the expression when $a = 5$ and $b = 3$.

5. $\left(\frac{a}{5} - a\right) - \left(\frac{b}{5} - b\right)$

6. $\left(6a - \frac{a^3}{3}\right) - \left(6b - \frac{b^3}{3}\right)$

In Exercises 7–10, integrate the marginal function.

7. $\frac{dC}{dx} = 0.02x^{3/2} + 29{,}500$

8. $\frac{dR}{dx} = 9000 + 2x$

9. $\frac{dP}{dx} = 25{,}000 - 0.01x$

10. $\frac{dC}{dx} = 0.03x^2 + 4600$

EXERCISES 5.4

In Exercises 1–8, sketch the region whose area is represented by the definite integral. Then use a geometric formula to evaluate the integral.

1. $\int_0^2 3\,dx$

2. $\int_0^4 2\,dx$

3. $\int_0^5 (x + 1)\,dx$

4. $\int_0^3 (2x + 1)\,dx$

5. $\int_{-2}^3 |x - 1|\,dx$

6. $\int_{-1}^4 |x - 2|\,dx$

7. $\int_{-3}^3 \sqrt{9 - x^2}\,dx$

8. $\int_0^2 \sqrt{4 - x^2}\,dx$

In Exercises 9 and 10, use the values $\int_0^5 f(x)\,dx = 8$ and $\int_0^5 g(x)\,dx = 3$ to evaluate the definite integral.

9. (a) $\int_0^5 [f(x) + g(x)]\,dx$ (b) $\int_0^5 [f(x) - g(x)]\,dx$

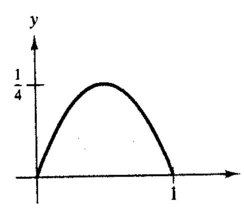
(c) $\int_0^5 -4f(x)\,dx$ (d) $\int_0^5 [f(x) - 3g(x)]\,dx$

10. (a) $\int_0^5 2g(x)\,dx$ (b) $\int_5^0 f(x)\,dx$

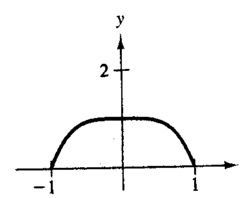
(c) $\int_5^5 f(x)\,dx$ (d) $\int_0^5 [f(x) - f(x)]\,dx$

In Exercises 11–18, find the area of the region.

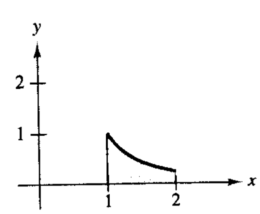
11. $y = x - x^2$



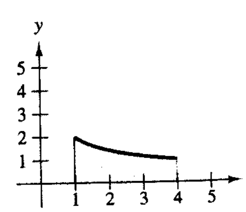
12. $y = 1 - x^4$



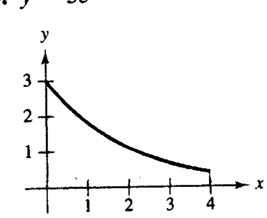
13. $y = \frac{1}{x^2}$



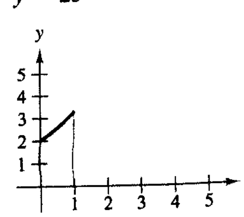
14. $y = \frac{2}{\sqrt{x}}$



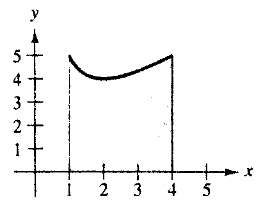
15. $y = 3e^{-x/2}$



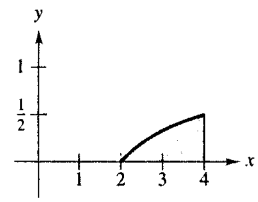
16. $y = 2e^{x/2}$



17. $y = \dfrac{x^2 + 4}{x}$



18. $y = \dfrac{x - 2}{x}$



In Exercises 19–42, evaluate the definite integral.

19. $\int_0^1 2x\,dx$

20. $\int_2^7 3v\,dv$

21. $\int_{-1}^0 (2x + 1)\,dx$

22. $\int_2^5 (-3x + 4)\,dx$

23. $\int_{-1}^1 (2t - 1)^2\,dt$

24. $\int_0^1 (1 - 2x)^2\,dx$

25. $\int_0^3 (x - 2)^3\,dx$

26. $\int_2^2 (x - 3)^4\,dx$

27. $\int_{-1}^1 (\sqrt[3]{t} - 2)\,dt$

28. $\int_1^4 \sqrt{\dfrac{2}{x}}\,dx$

29. $\int_1^4 \dfrac{2u - 1}{\sqrt{u}}\,du$

30. $\int_0^1 \dfrac{x - \sqrt{x}}{3}\,dx$

31. $\int_{-1}^0 (t^{1/3} - t^{2/3})\,dt$

32. $\int_0^4 (x^{1/2} + x^{1/4})\,dx$

33. $\int_0^4 \dfrac{1}{\sqrt{2x + 1}}\,dx$

34. $\int_0^2 \dfrac{x}{\sqrt{1 + 2x^2}}\,dx$

35. $\int_0^1 e^{-2x}\,dx$

36. $\int_1^2 e^{1-x}\,dx$

37. $\int_1^3 \dfrac{e^{3/x}}{x^2}\,dx$

38. $\int_{-1}^1 (e^x - e^{-x})\,dx$

39. $\int_0^1 e^{2x}\sqrt{e^{2x} + 1}\,dx$

40. $\int_0^1 \dfrac{e^{-x}}{\sqrt{e^{-x} + 1}}\,dx$

41. $\int_0^2 \dfrac{x}{1 + 4x^2}\,dx$

42. $\int_0^1 \dfrac{e^{2x}}{e^{2x} + 1}\,dx$

In Exercises 43–46, evaluate the definite integral by the most convenient method. Explain your approach.

43. $\int_{-1}^1 |4x|\,dx$

44. $\int_0^3 |2x - 3|\,dx$

45. $\int_0^4 (2 - |x - 2|)\,dx$

46. $\int_{-4}^4 (4 - |x|)\,dx$

In Exercises 47–50, evaluate the definite integral by hand. Then use a symbolic integration utility to evaluate the definite integral. Briefly explain any differences in your results.

47. $\int_{-1}^2 \dfrac{x}{x^2 - 9}\,dx$

48. $\int_2^3 \dfrac{x + 1}{x^2 + 2x - 3}\,dx$

49. $\int_0^3 \dfrac{2e^x}{2 + e^x}\,dx$

50. $\int_1^2 \dfrac{(2 + \ln x)^3}{x}\,dx$

In Exercises 51–56, evaluate the definite integral by hand. Then use a graphing utility to graph the region whose area is represented by the integral.

51. $\int_1^3 (4x - 3)\,dx$

52. $\int_0^2 (x + 4)\,dx$

53. $\int_0^1 (x - x^3)\,dx$

54. $\int_0^1 \sqrt{x}(1 - x)\,dx$

55. $\int_2^4 \dfrac{3x^2}{x^3 - 1}\,dx$

56. $\int_0^{\ln 6} \dfrac{e^x}{2}\,dx$

In Exercises 57–60, find the area of the region bounded by the graphs. Use a graphing utility to graph the region and verify your results.

57. $y = 3x^2 + 1$, $y = 0$, $x = 0$, and $x = 2$

58. $y = 1 + \sqrt{x}$, $y = 0$, $x = 0$, and $x = 4$

59. $y = (x + 5)/x$, $y = 0$, $x = 1$, and $x = 5$

60. $y = 3e^x$, $y = 0$, $x = -2$, and $x = 1$

In Exercises 61–68, use a graphing utility to graph the function over the interval. Find the average value of the function over the interval. Then find all x-values in the interval for which the function is equal to its average value.

Function	*Interval*
61. $f(x) = 6 - x^2$	$[-2, 2]$
62. $f(x) = x - 2\sqrt{x}$	$[0, 4]$
63. $f(x) = 5e^{0.2(x-10)}$	$[0, 10]$
64. $f(x) = 2e^{x/4}$	$[0, 4]$
65. $f(x) = x\sqrt{4 - x^2}$	$[0, 2]$
66. $f(x) = \dfrac{1}{(x - 3)^2}$	$[0, 2]$
67. $f(x) = \dfrac{5x}{x^2 + 1}$	$[0, 7]$
68. $f(x) = \dfrac{4x}{x^2 + 1}$	$[0, 1]$

In Exercises 69–72, state whether the function is even, odd, or neither.

69. $f(x) = 3x^4$

70. $g(x) = x^3 - 2x$

71. $g(t) = 2t^5 - 3t^2$

72. $f(t) = 5t^4 + 1$

73. Use the value $\int_0^2 x^2\,dx = \frac{8}{3}$ to evaluate each definite integral. Explain your reasoning.

(a) $\int_{-2}^0 x^2\,dx$ (b) $\int_{-2}^2 x^2\,dx$ (c) $\int_0^2 -x^2\,dx$

74. Use the value $\int_0^2 x^3\,dx = 4$ to evaluate each definite integral. Explain your reasoning.

(a) $\int_{-2}^0 x^3\,dx$ (b) $\int_{-2}^2 x^3\,dx$ (c) $\int_0^2 3x^3\,dx$

Marginal Analysi in cost C, revenue R, … case, assume that the … specified value of x.

Marginal

75. $\dfrac{dC}{dx} = 2.25$

76. $\dfrac{dC}{dx} = \dfrac{20{,}000}{x^2}$

77. $\dfrac{dR}{dx} = 48 - 3x$

78. $\dfrac{dR}{dx} = 75(20 + $ …

79. $\dfrac{dP}{dx} = \dfrac{400 - x}{150}$

80. $\dfrac{dP}{dx} = 12.5(40 -$ …

Annuity In Exercise … with income function …

81. $c(t) = \$250$, r …

82. $c(t) = \$500$, r …

83. $c(t) = \$1500$, r …

84. $c(t) = \$2000$, r …

Capital Accumulati … rate of investment $dI/$ … five-year period by eva…

Capital accumulat…

where t is the time in y…

85. $\dfrac{dI}{dt} = 500$

86. $\dfrac{dI}{dt} = 100t$

87. $\dfrac{dI}{dt} = 500\sqrt{t + 1}$

88. $\dfrac{dI}{dt} = \dfrac{12{,}000t}{(t^2 + 2)^2}$

89. ***Cost*** The total c… of equipment for …

$C = 5000(25$ …

Find the total cost … years.

Marginal Analysis In Exercises 75–80, find the change in cost C, revenue R, or profit P, for the given marginal. In each case, assume that the number of units x increases by 3 from the specified value of x.

Marginal	*Number of Units, x*
75. $\frac{dC}{dx} = 2.25$	$x = 100$
76. $\frac{dC}{dx} = \frac{20{,}000}{x^2}$	$x = 10$
77. $\frac{dR}{dx} = 48 - 3x$	$x = 12$
78. $\frac{dR}{dx} = 75\left(20 + \frac{900}{x}\right)$	$x = 500$
79. $\frac{dP}{dx} = \frac{400 - x}{150}$	$x = 200$
80. $\frac{dP}{dx} = 12.5\left(40 - 3\sqrt{x}\right)$	$x = 125$

Annuity In Exercises 81–84, find the amount of an annuity with income function $c(t)$, interest rate r, and term T.

81. $c(t) = \$250, \quad r = 8\%, \quad T = 6$ years

82. $c(t) = \$500, \quad r = 9\%, \quad T = 4$ years

83. $c(t) = \$1500, \quad r = 2\%, \quad T = 10$ years

84. $c(t) = \$2000, \quad r = 3\%, \quad T = 15$ years

Capital Accumulation In Exercises 85–88, you are given the rate of investment dI/dt. Find the capital accumulation over a five-year period by evaluating the definite integral

$$\text{Capital accumulation} = \int_0^5 \frac{dI}{dt}\,dt$$

where t is the time in years.

85. $\frac{dI}{dt} = 500$

86. $\frac{dI}{dt} = 100t$

87. $\frac{dI}{dt} = 500\sqrt{t + 1}$

88. $\frac{dI}{dt} = \frac{12{,}000t}{(t^2 + 2)^2}$

89. ***Cost*** The total cost of purchasing and maintaining a piece of equipment for x years can be modeled by

$$C = 5000\left(25 + 3\int_0^x t^{1/4}\,dt\right).$$

Find the total cost after (a) 1 year, (b) 5 years, and (c) 10 years.

90. ***Depreciation*** A company purchases a new machine for which the rate of depreciation can be modeled by

$$\frac{dV}{dt} = 10{,}000(t - 6), \quad 0 \le t \le 5$$

where V is the value of the machine after t years. Set up and evaluate the definite integral that yields the total loss of value of the machine over the first 3 years.

91. ***Compound Interest*** A deposit of \$2250 is made in a savings account at an annual interest rate of 12%, compounded continuously. Find the average balance in the account during the first 5 years.

92. ***Mortgage Debt*** The rate of change of mortgage debt outstanding for one- to four-family homes in the United States from 1993 through 2002 can be modeled by

$$\frac{dM}{dt} = 5.4399t^2 + 6603.7e^{-t}$$

where M is the mortgage debt outstanding (in billions of dollars) and $t = 3$ corresponds to 1993. In 1993, the mortgage debt outstanding in the United States was \$3119 billion. *(Source: Board of Governors of the Federal Reserve System)*

(a) Write a model for the debt as a function of t.

(b) What was the average mortgage debt outstanding for 1993 through 2002?

93. ***Medicine*** The velocity v of blood at a distance r from the center of an artery of radius R can be modeled by

$$v = k(R^2 - r^2)$$

where k is a constant. Find the average velocity along a radius of the artery. (Use 0 and R as the limits of integration.)

94. ***Biology*** The rate of change in the number of coyotes $N(t)$ in a population is directly proportional to $650 - N(t)$, where t is time in years.

$$\frac{dN}{dt} = k[650 - N(t)]$$

When $t = 0$, the population is 300, and when $t = 2$, the population has increased to 500.

(a) Find the population function.

(b) Find the average number of coyotes over the first 5 years.

In Exercises 95–98, use a symbolic integration utility to evaluate the definite integral.

95. $\int_3^6 \frac{x}{3\sqrt{x^2 - 8}}\,dx$

96. $\int_{1/2}^1 (x + 1)\sqrt{1 - x}\,dx$

97. $\int_2^5 \left(\frac{1}{x^2} - \frac{1}{x^3}\right)dx$

98. $\int_0^1 x^3(x^3 + 1)^3\,dx$

PREREQUISITE REVIEW 5.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, simplify the expression.

1. $(-x^2 + 4x + 3) - (x + 1)$
2. $(-2x^2 + 3x + 9) - (-x + 5)$
3. $(-x^3 + 3x^2 - 1) - (x^2 - 4x + 4)$
4. $(3x + 1) - (-x^3 + 9x + 2)$

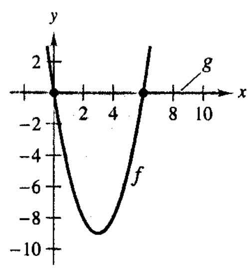
In Exercises 5–10, find the points of intersection of the graphs.

5. $f(x) = x^2 - 4x + 4,\ g(x) = 4$
6. $f(x) = -3x^2,\ g(x) = 6 - 9x$
7. $f(x) = x^2,\ g(x) = -x + 6$
8. $f(x) = \frac{1}{2}x^3,\ g(x) = 2x$
9. $f(x) = x^2 - 3x,\ g(x) = 3x - 5$
10. $f(x) = e^x,\ g(x) = e$

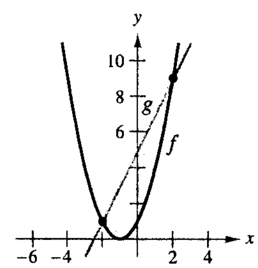
EXERCISES 5.5

In Exercises 1–8, find the area of the region.

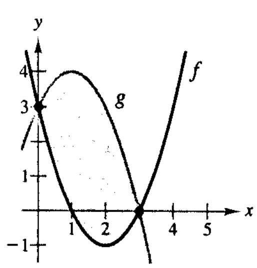
1. $f(x) = x^2 - 6x$
   $g(x) = 0$



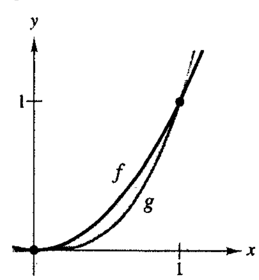
2. $f(x) = x^2 + 2x + 1$
   $g(x) = 2x + 5$



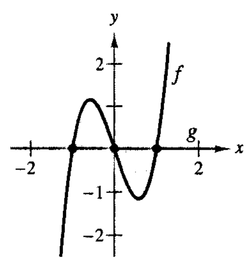
3. $f(x) = x^2 - 4x + 3$
   $g(x) = -x^2 + 2x + 3$



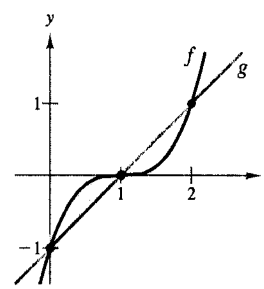
4. $f(x) = x^2$
   $g(x) = x^3$



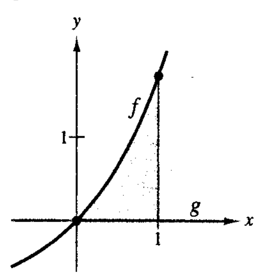
5. $f(x) = 3(x^3 - x)$
   $g(x) = 0$



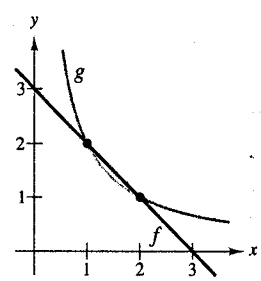
6. $f(x) = (x - 1)^3$
   $g(x) = x - 1$



7. $f(x) = e^x - 1$
   $g(x) = 0$



8. $f(x) = -x + 3$
   $g(x) = 2x^{-1}$



In Exercises 9–14, sk
the definite integra

9. $\int_0^4 [(x + 1) -$
10. $\int_{-1}^1 [(1 - x^2)$
11. $\int_{-2}^2 [2x^2 - (x^4$
12. $\int_{-4}^0 [(x - 6) -$
13. $\int_{-1}^2 [(y^2 + 2)$
14. $\int_{-2}^3 [(y + 6) -$

In Exercises 15–26, s
the functions and fi

15. $y = \frac{1}{x^2}, y = 0,$
16. $y = x^3 - 2x +$
17. $f(x) = \sqrt[3]{x}, g(x$
18. $f(x) = \sqrt{3x} +$
19. $y = x^2 - 4x +$
20. $y = 4 - x^2, y =$
21. $y = xe^{-x^2}, y =$
22. $y = \frac{e^{1/x}}{x^2}, y = 0$
23. $y = \frac{8}{x}, y = x^2,$
24. $y = \frac{1}{x}, y = x^3,$
25. $f(x) = e^{0.5x}, g($
26. $f(x) = \frac{1}{x}, g(x)$

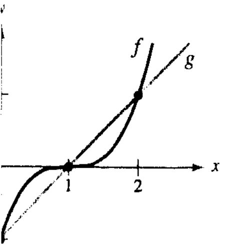
In Exercises 27–30, sl
the functions and fin

27. $f(y) = y^2, g(y)$
28. $f(y) = y(2 - y$
29. $f(y) = \sqrt{y}, y =$
30. $f(y) = y^2 + 1,$

r sections. You will

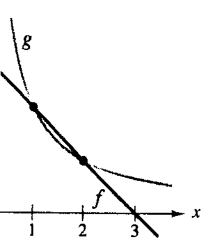
$) = (x - 1)^3$

$) = x - 1$



$| = -x + 3$

$= 2x^{-1}$



In Exercises 9–14, sketch the region whose area is represented by the definite integral.

9. $\int_0^4 \left[(x+1) - \frac{1}{2}x\right] dx$

10. $\int_{-1}^1 [(1 - x^2) - (x^2 - 1)] \, dx$

11. $\int_{-2}^2 [2x^2 - (x^4 - 2x^2)] \, dx$

12. $\int_{-4}^0 [(x - 6) - (x^2 + 5x - 6)] \, dx$

13. $\int_{-1}^2 [(y^2 + 2) - 1] \, dy$

14. $\int_{-2}^3 [(y + 6) - y^2] \, dy$

In Exercises 15–26, sketch the region bounded by the graphs of the functions and find the area of the region.

15. $y = \dfrac{1}{x^2}, y = 0, x = 1, x = 5$

16. $y = x^3 - 2x + 1, y = -2x, x = 1$

17. $f(x) = \sqrt[3]{x}, g(x) = x$

18. $f(x) = \sqrt{3x} + 1, g(x) = x + 1$

19. $y = x^2 - 4x + 3, y = 3 + 4x - x^2$

20. $y = 4 - x^2, y = x^2$

21. $y = xe^{-x^2}, y = 0, x = 0, x = 1$

22. $y = \dfrac{e^{1/x}}{x^2}, y = 0, x = 1, x = 3$

23. $y = \dfrac{8}{x}, y = x^2, y = 0, x = 1, x = 4$

24. $y = \dfrac{1}{x}, y = x^3, x = \dfrac{1}{2}, x = 1$

25. $f(x) = e^{0.5x}, g(x) = -\dfrac{1}{x}, x = 1, x = 2$

26. $f(x) = \dfrac{1}{x}, g(x) = -e^x, x = \dfrac{1}{2}, x = 1$

In Exercises 27–30, sketch the region bounded by the graphs of the functions and find the area of the region.

27. $f(y) = y^2, g(y) = y + 2$

28. $f(y) = y(2 - y), g(y) = -y$

29. $f(y) = \sqrt{y}, y = 9, x = 0$

30. $f(y) = y^2 + 1, g(y) = 4 - 2y$

In Exercises 31–34, use a graphing utility to graph the region bounded by the graphs of the functions. Write the definite integrals that represent the area of the region. (*Hint:* Multiple integrals may be necessary.)

31. $f(x) = 2x, \; g(x) = 4 - 2x, \; h(x) = 0$

32. $f(x) = x(x^2 - 3x + 3), g(x) = x^2$

33. $y = \dfrac{4}{x}, y = x, x = 1, x = 4$

34. $y = x^3 - 4x^2 + 1, y = x - 3$

In Exercises 35–38, use a graphing utility to graph the region bounded by the graphs of the functions, and find the area of the region.

35. $f(x) = x^2 - 4x, g(x) = 0$

36. $f(x) = 3 - 2x - x^2, g(x) = 0$

37. $f(x) = x^2 + 2x + 1, g(x) = x + 1$

38. $f(x) = -x^2 + 4x + 2, g(x) = x + 2$

In Exercises 39 and 40, use integration to find the area of the triangular region having the given vertices.

39. $(0, 0), (4, 0), (4, 4)$

40. $(0, 0), (4, 0), (6, 4)$

Consumer and Producer Surpluses In Exercises 41–46, find the consumer and producer surpluses.

	Demand Function	*Supply Function*
41.	$p_1(x) = 50 - 0.5x$	$p_2(x) = 0.125x$
42.	$p_1(x) = 300 - x$	$p_2(x) = 100 + x$
43.	$p_1(x) = 200 - 0.02x^2$	$p_2(x) = 100 + x$
44.	$p_1(x) = 1000 - 0.4x^2$	$p_2(x) = 42x$
45.	$p_1(x) = \dfrac{10{,}000}{\sqrt{x + 100}}$	$p_2(x) = 100\sqrt{0.05x + 10}$
46.	$p_1(x) = \sqrt{25 - 0.1x}$	$p_2(x) = \sqrt{9 + 0.1x} - 2$

47. ***Writing*** Describe the characteristics of typical demand and supply functions.

48. ***Writing*** Suppose that the demand and supply functions for a product do not intersect. What can you conclude?

Revenue In Exercises 49 and 50, two models, R_1 and R_2, are given for revenue (in billions of dollars per year) for a large corporation. Both models are estimates of revenues for 2004–2008, with $t = 4$ corresponding to 2004. Which model is projecting the greater revenue? How much more total revenue does that model project over the four-year period?

49. $R_1 = 7.21 + 0.58t, R_2 = 7.21 + 0.45t$

50. $R_1 = 7.21 + 0.26t + 0.02t^2, R_2 = 7.21 + 0.1t + 0.01t^2$

51. ***Fuel Cost*** The projected fuel cost C (in millions of dollars per year) for an airline company from 2004 through 2010 is $C_1 = 568.5 + 7.15t$, where $t = 4$ corresponds to 2004. If the company purchases more efficient airplane engines, fuel cost is expected to decrease and to follow the model $C_2 = 525.6 + 6.43t$. How much can the company save with the more efficient engines? Explain your reasoning.

52. ***Health*** An epidemic was spreading such that t weeks after its outbreak it had infected

$$N_1(t) = 0.1t^2 + 0.5t + 150, \quad 0 \le t \le 50$$

people. Twenty-five weeks after the outbreak, a vaccine was developed and administered to the public. At that point, the number of people infected was governed by the model

$$N_2(t) = -0.2t^2 + 6t + 200.$$

Approximate the number of people that the vaccine prevented from becoming ill during the epidemic.

53. ***Consumer Trends*** For the years 1990 through 2001, the per capita consumption of tomatoes (in pounds per year) in the United States can be modeled by

$$C(t) = \begin{cases} 0.085t^3 - 0.309t^2 + 0.13t + 15.5, \\ 0 \le t \le 4 \\ 0.01515t^4 - 0.5348t^3 + 6.864t^2 - 37.68t + 91.4 \\ 4 < t \le 11 \end{cases}$$

where $t = 0$ corresponds to 1990. *(Source: U.S. Department of Agriculture)*

(a) Use a graphing utility to graph this model.

(b) Suppose the tomato consumption from 1995 through 2001 had continued to follow the model for 1990 through 1994. How many more or fewer pounds of tomatoes would have been consumed from 1995 through 2001?

54. ***Consumer and Producer Surpluses*** Factory orders for an air conditioner are about 6000 units per week when the price is \$331 and about 8000 units per week when the price is \$303. The supply function is given by $p = 0.0275x$. Find the consumer and producer surpluses. (Assume the demand function is linear.)

55. ***Consumer and Producer Surpluses*** Repeat Exercise 54 with a demand of about 6000 units per week when the price is \$325 and about 8000 units per week when the price is \$300. Find the consumer and producer surpluses. (Assume the demand function is linear.)

56. ***Cost, Revenue, and Profit*** The revenue from a manufacturing process (in millions of dollars per year) is projected to follow the model $R = 100$ for 10 years. Over the same period of time, the cost (in millions of dollars per year) is projected to follow the model $C = 60 + 0.2t^2$, where t is the time (in years). Approximate the profit over the 10-year period.

57. ***Cost, Revenue, and Profit*** Repeat Exercise 56 for revenue and cost models given by $R = 100 + 0.08t$ and $C = 60 + 0.2t^2$.

58. ***Lorenz Curve*** Economists use *Lorenz curves* to illustrate the distribution of income in a country. Letting x represent the percent of families in a country and y the percent of total income, the model $y = x$ would represent a country in which each family had the same income. The Lorenz curve, $y = f(x)$, represents the actual income distribution. The area between these two models, for $0 \le x \le 100$, indicates the "income inequality" of a country. In 2001, the Lorenz curve for the United States could be modeled by

$$y = (0.00059x^2 + 0.0233x + 1.731)^2, \quad 0 \le x \le 100$$

where x is measured from the poorest to the wealthiest families. Find the income inequality for the United States in 2001. *(Source: U.S. Census Bureau)*

59. ***Income Distribution*** Using the Lorenz curve in Exercise 58, complete the table, which lists the percent of total income earned by each quintile in the United States in 2001.

Quintile	Lowest	2nd	3rd	4th	Highest
Percent					

BUSINESS CAPSULE

EricCrossan.com

In 1994, wardrobe consultant and personal shopper Marilyn N. Wright started the company Marilyn's Fashions in Newark, Delaware. Wright consults with her 450 American and 250 international clients on their wardrobes, styles, and budgets, then delivers the clothes and accessories to their doorsteps in less than 2 weeks. She used \$5000 in start-up capital while working as a claims adjuster and now brings in \$250,000 in annual revenue.

60. ***Research Project*** Use your school's library, the Internet, or some other reference source to research a small company similar to that described above. Describe the impact of different factors, such as start-up capital and market conditions, on a company's revenue.

5.(

The Midpoint

In Section 5.4, you
Calculus to evaluate
integrand. In cases
the integral using a
Midpoint Rule. (T

EXAMPLE 1

Use the five rectan
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$x = 2$.

SOLUTION You ca
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Evaluate

The width of each r

Area ≈

$= 7.36.$

TRY IT 1

Use four rectangles
graph of $f(x) = x^2$

For the region in
gral. That is,

Area =

PREREQUISITE REVIEW 5.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find the midpoint of the interval.

1. $\left[0, \frac{1}{3}\right]$

2. $\left[\frac{1}{10}, \frac{2}{10}\right]$

3. $\left[\frac{3}{20}, \frac{4}{20}\right]$

4. $\left[1, \frac{7}{6}\right]$

5. $\left[2, \frac{31}{15}\right]$

6. $\left[\frac{26}{9}, 3\right]$

In Exercises 7–10, find the limit.

7. $\lim_{x \to \infty} \dfrac{2x^2 + 4x - 1}{3x^2 - 2x}$

8. $\lim_{x \to \infty} \dfrac{4x + 5}{7x - 5}$

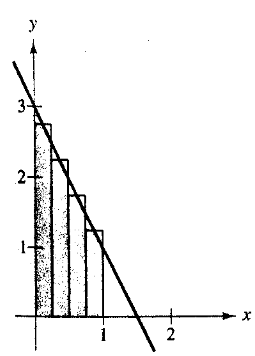
9. $\lim_{x \to \infty} \dfrac{x - 7}{x^2 + 1}$

10. $\lim_{x \to \infty} \dfrac{5x^3 + 1}{x^3 + x^2 + 4}$

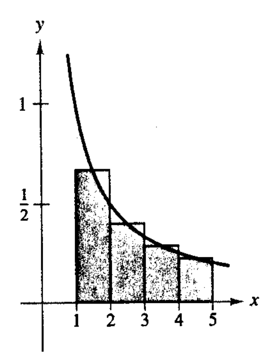
EXERCISES 5.6

In Exercises 1–4, use the Midpoint Rule with $n = 4$ to approximate the area of the region. Compare your result with the exact area obtained with a definite integral.

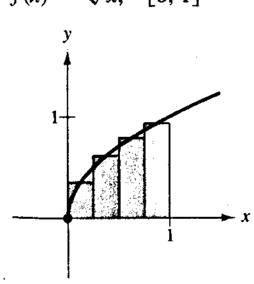
1. $f(x) = -2x + 3, \quad [0, 1]$



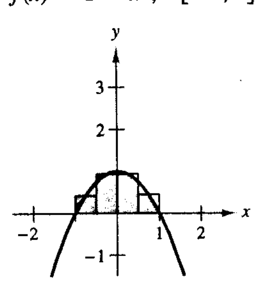
2. $f(x) = \dfrac{1}{x}, \quad [1, 5]$



3. $f(x) = \sqrt{x}, \quad [0, 1]$



4. $f(x) = 1 - x^2, \quad [-1, 1]$



In Exercises 5–12, use the Midpoint Rule with $n = 4$ to approximate the area of the region bounded by the graph of f and the x-axis over the interval. Compare your result with the exact area. Sketch the region.

Function	*Interval*
5. $f(x) = x^2 + 2$	$[-1, 1]$
6. $f(x) = 4 - x^2$	$[0, 2]$
7. $f(x) = 2x^2$	$[1, 3]$
8. $f(x) = 2x - x^3$	$[0, 1]$
9. $f(x) = x^2 - x^3$	$[0, 1]$
10. $f(x) = x^2 - x^3$	$[-1, 0]$
11. $f(x) = x(1 - x)^2$	$[0, 1]$
12. $f(x) = x^2(3 - x)$	$[0, 3]$

In Exercises 13–16, use a program similar to that on page 366 to approximate the area of the region. How large must n be to obtain an approximation that is correct to within 0.01?

13. $\displaystyle\int_0^4 (2x^2 + 3)\,dx$

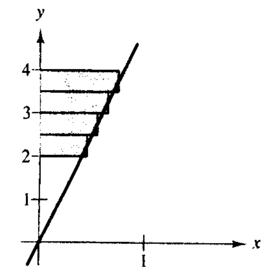
14. $\displaystyle\int_0^4 (2x^3 + 3)\,dx$

15. $\displaystyle\int_1^2 (2x^2 - x + 1)\,dx$

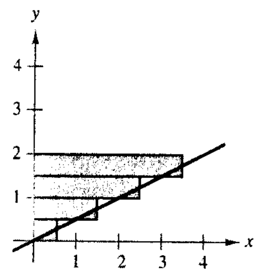
16. $\displaystyle\int_1^2 (x^3 - 1)\,dx$

In Exercises 17–20, use the Midpoint Rule with $n = 4$ to approximate the area of the region. Compare your result with the exact area obtained with a definite integral.

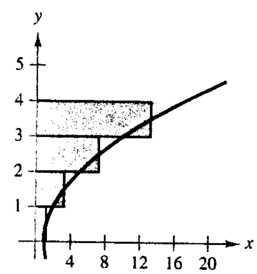
17. $f(y) = \frac{1}{4}y, \quad [2, 4]$



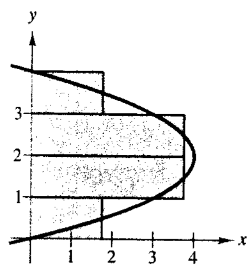
18. $f(y) = 2y, \quad [0, 2]$



19. $f(y) = y^2 + 1, \quad [0, 4]$

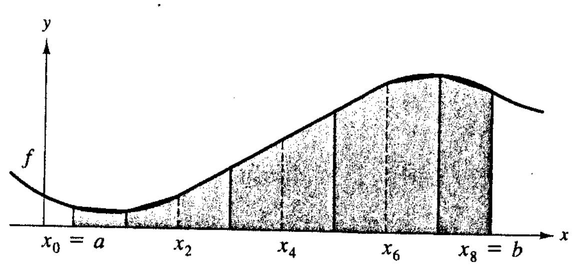


20. $f(y) = 4y - y^2, \quad [0, 4]$



Trapezoidal Rule In Exercises 21 and 22, use the Trapezoidal Rule with $n = 8$ to approximate the definite integral. Compare the result with the exact value and the approximation obtained with $n = 8$ and the Midpoint Rule. Which approximation technique appears to be better? Let f be continuous on $[a, b]$ and let n be the number of equal subintervals (see figure). Then the Trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is

$$\frac{b-a}{2n}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)].$$



21. $\int_0^2 x^3\,dx$

22. $\int_1^3 \frac{1}{x^2}\,dx$

In Exercises 23–26, use the Trapezoidal Rule with $n = 4$ to approximate the definite integral.

23. $\int_0^2 \frac{1}{x+1}\,dx$

24. $\int_0^4 \sqrt{1+x^2}\,dx$

25. $\int_{-1}^1 \frac{1}{x^2+1}\,dx$

26. $\int_1^5 \frac{\sqrt{x-1}}{x}\,dx$

In Exercises 27 and 28, use a computer or programmable calculator to approximate the definite integral using the Midpoint Rule and the Trapezoidal Rule for $n = 4, 8, 12, 16$, and 20.

27. $\int_0^4 \sqrt{2+3x^2}\,dx$

28. $\int_0^2 \frac{5}{x^3+1}\,dx$

In Exercises 29 and 30, use the Trapezoidal Rule with $n = 10$ to approximate the area of the region bounded by the graphs of the equations.

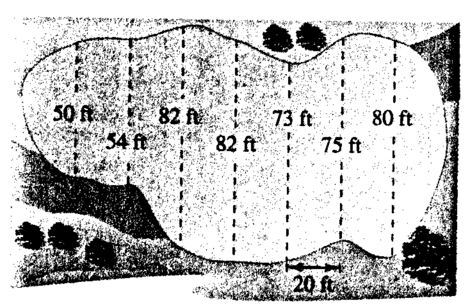
29. $y = \sqrt{\frac{x^3}{4-x}}, \quad y = 0, \quad x = 3$

30. $y = x\sqrt{\frac{4-x}{4+x}}, \quad y = 0, \quad x = 4$

31. ***Velocity and Acceleration*** The table lists the velocity v (in feet per second) of an accelerating car over a 20-second interval. Use the Trapezoidal Rule to approximate the distance in feet that the car travels during the 20 seconds. (The distance is given by $s = \int_0^{20} v\,dt$.)

Time, t	0	5	10	15	20
Velocity, v	0.0	29.3	51.3	66.0	73.3

32. ***Surface Area*** To estimate the surface area of a pond, a surveyor takes several measurements, as shown in the figure. Estimate the surface area of the pond using (a) the Midpoint Rule and (b) the Trapezoidal Rule.



33. ***Numerical Approximation*** Use the Midpoint Rule and the Trapezoidal Rule with $n = 4$ to approximate π where

$$\pi = \int_0^1 \frac{4}{1+x^2}\,dx.$$

Then use a graphing utility to evaluate the definite integral. Compare all of your results.

5.

The Disk Me

Another importan volume of a three type of three-dim begin with solids and pistons, are us

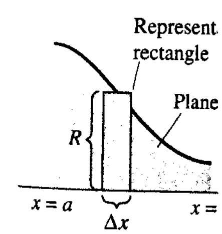
As shown in plane region abou

To develop a f a continuous funct area of the region i Figure 5.26. By re disks, each with a revolving the regi volumes of the n di can see that the ex the **Disk Method.**

The Disk Meth

The volume of graph of f and

Volume =



Approximation by n r

FIGURE 5.26

PREREQUISITE REVIEW 5.7

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, solve for x.

1. $x^2 = 2x$
2. $-x^2 + 4x = x^2$
3. $x = -x^3 + 5x$
4. $x^2 + 1 = x + 3$
5. $-x + 4 = \sqrt{4x - x^2}$
6. $\sqrt{x - 1} = \frac{1}{2}(x - 1)$

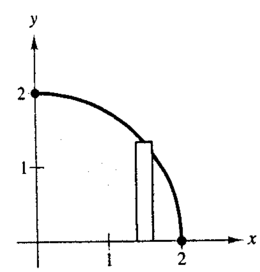
In Exercises 7–10, evaluate the integral.

7. $\int_0^2 2e^{2x}\,dx$
8. $\int_{-1}^3 \frac{2x + 1}{x^2 + x + 2}\,dx$
9. $\int_0^2 x\sqrt{x^2 + 1}\,dx$
10. $\int_1^5 \frac{(\ln x)^2}{x}\,dx$

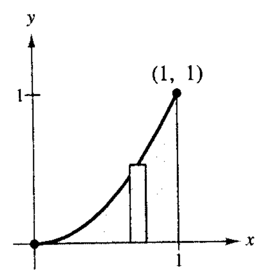
EXERCISES 5.7

In Exercises 1–16, find the volume of the solid formed by revolving the region bounded by the graph(s) of the equation(s) about the x-axis.

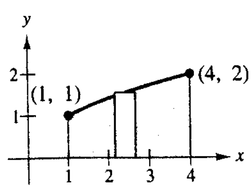
1. $y = \sqrt{4 - x^2}$



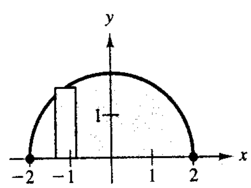
2. $y = x^2$



3. $y = \sqrt{x}$



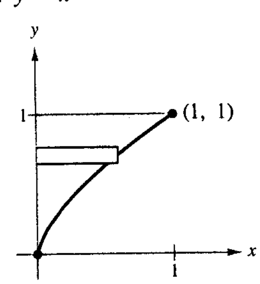
4. $y = \sqrt{4 - x^2}$



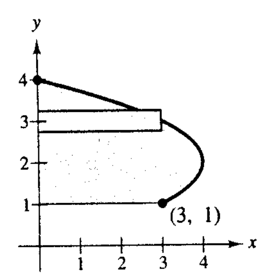
5. $y = 4 - x^2, \quad y = 0$
6. $y = x, \quad y = 0, \quad x = 4$
7. $y = 1 - \frac{1}{4}x^2, \quad y = 0$
8. $y = x^2 + 1, \quad y = 5$
9. $y = -x + 1, \quad y = 0, \quad x = 0$
10. $y = x, \quad y = e^{x-1}, \quad x = 0$
11. $y = \sqrt{x} + 1, \quad y = 0, \quad x = 0, \quad x = 9$
12. $y = \sqrt{x}, \quad y = 0, \quad x = 4$
13. $y = 2x^2, \quad y = 0, \quad x = 2$
14. $y = \frac{1}{x}, \quad y = 0, \quad x = 1, \quad x = 3$
15. $y = e^x, \quad y = 0, \quad x = 0, \quad x = 1$
16. $y = x^2, \quad y = 4x - x^2$

In Exercises 17–24, find the volume of the solid formed by revolving the region bounded by the graph(s) of the equation(s) about the y-axis.

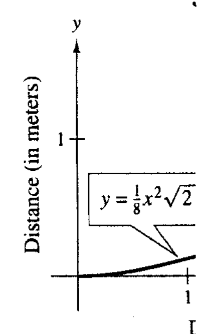
17. $y = x^2, \quad y = 4, \quad 0 \le x \le 2$
18. $y = \sqrt{16 - x^2}, \quad y = 0, \quad 0 \le x \le 4$
19. $x = 1 - \frac{1}{2}y, \quad x = 0, \quad y = 0$
20. $x = y(y - 1), \quad x = 0$
21. $y = x^{2/3}$



22. $x = -y^2 + 4y$



23. $y = \sqrt{4 - x}, \quad y =$
24. $y = 4, \quad y = 0, \quad x$
25. ***Volume*** The lin revolved about the volume of the cone?
26. ***Volume*** The lin revolved about the volume of the cone?
27. ***Volume*** Use the I of a right circular co base and h is the he
28. ***Volume*** Use the I of a sphere of radiu
29. ***Volume*** The right $9x^2 + 25y^2 = 2$ is revolved about t (shaped like an M spheroid.
30. ***Volume*** The upp $9x^2 + 16y^2 = 1$ is revolved about t (shaped like a footb
31. ***Volume*** A tank o by revolving the $y = \frac{1}{8}x^2\sqrt{2 - x}$ an and y are measured of the tank.



32. ***Volume*** A soup revolution formed

$y = \sqrt{\frac{x}{2} + 1},$

about the x-axis. measured in inches

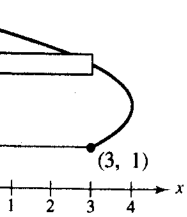
ections. You will

= 9

f the solid formed by
ph(s) of the equation(s)

4

$-y^2 + 4y$



23. $y = \sqrt{4 - x}, \quad y = 0, \quad x = 0$

24. $y = 4, \quad y = 0, \quad x = 2, \quad x = 0$

25. ***Volume*** The line segment from $(0, 0)$ to $(6, 3)$ is revolved about the x-axis to form a cone. What is the volume of the cone?

26. ***Volume*** The line segment from $(0, 0)$ to $(4, 2)$ is revolved about the y-axis to form a cone. What is the volume of the cone?

27. ***Volume*** Use the Disk Method to verify that the volume of a right circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height.

28. ***Volume*** Use the Disk Method to verify that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

29. ***Volume*** The right half of the ellipse

$$9x^2 + 25y^2 = 225$$

is revolved about the y-axis to form an oblate spheroid (shaped like an M&M candy). Find the volume of the spheroid.

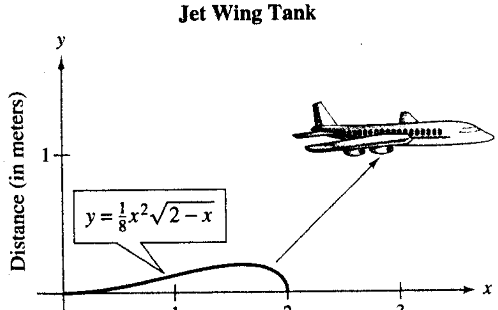
30. ***Volume*** The upper half of the ellipse

$$9x^2 + 16y^2 = 144$$

is revolved about the x-axis to form a prolate spheroid (shaped like a football). Find the volume of the spheroid.

31. ***Volume*** A tank on the wing of a jet airplane is modeled by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2 - x}$ and the x-axis about the x-axis, where x and y are measured in meters (see figure). Find the volume of the tank.

Jet Wing Tank



32. ***Volume*** A soup bowl can be modeled as a solid of revolution formed by revolving the graph of

$$y = \sqrt{\frac{x}{2}} + 1, \quad 0 \le x \le 4$$

about the x-axis. Use this model, where x and y are measured in inches, to find the volume of the soup bowl.

33. ***Biology*** A pond is to be stocked with a species of fish. The food supply in 500 cubic feet of pond water can adequately support one fish. The pond is nearly circular, is 20 feet deep at its center, and has a radius of 200 feet. The bottom of the pond can be modeled by

$$y = 20[(0.005x)^2 - 1].$$

(a) How much water is in the pond?

(b) How many fish can the pond support?

34. ***Modeling a Body of Water*** A pond is approximately circular, with a diameter of 400 feet (see figure). Starting at the center, the depth of the water is measured every 25 feet and recorded in the table.

x	0	25	50	75	100
Depth	20	19	19	17	15

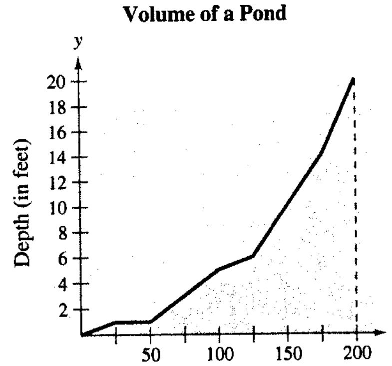
x	125	150	175	200
Depth	14	10	6	0

(a) Use a graphing utility to plot the depths and graph the model of the pond's depth, $y = 20 - 0.00045x^2$.

(b) Use the model in part (a) to find the pond's volume.

(c) Use the result of part (b) to approximate the number of gallons of water in the pond ($1 \text{ ft}^3 \approx 7.48$ gal).

Volume of a Pond



In Exercises 35 and 36, use a program similar to the one on page 366 to approximate the volume of a solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

35. $y = \sqrt[3]{x + 1}, \quad y = 0, \quad x = 0, \quad x = 7$

36. $y = \dfrac{10}{x^2 + 1}, \quad y = 0, \quad x = 0, \quad x = 3$