PREREQUISITE REVIEW 6.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, evaluate the indefinite integral.

1. $\int 5\, dx$

2. $\int \frac{1}{3}\, dx$

3. $\int x^{3/2}\, dx$

4. $\int x^{2/3}\, dx$

5. $\int 2x(x^2+1)^3\, dx$

6. $\int 3x^2(x^3-1)^2\, dx$

7. $\int 6e^{6x}\, dx$

8. $\int \frac{2}{2x+1}\, dx$

In Exercises 9–12, simplify the expression.

9. $2x(x-1)^2 + x(x-1)$

10. $6x(x+4)^3 - 3x^2(x+4)^2$

11. $3(x+7)^{1/2} - 2x(x+7)^{-1/2}$

12. $(x+5)^{1/3} - 5(x+5)^{-2/3}$

EXERCISES 6.1

In Exercises 1–38, find the indefinite integral.

1. $\int (x-2)^4\, dx$

2. $\int (x+5)^{3/2}\, dx$

3. $\int \frac{2}{(t-9)^2}\, dt$

4. $\int \frac{4}{(1-t)^3}\, dt$

5. $\int \frac{2t-1}{t^2-t+2}\, dt$

6. $\int \frac{2y^3}{y^4+1}\, dy$

7. $\int \sqrt{1+x}\, dx$

8. $\int (3+x)^{5/2}\, dx$

9. $\int \frac{12x+2}{3x^2+x}\, dx$

10. $\int \frac{6x^2+2}{x^3+x}\, dx$

11. $\int \frac{1}{(5x+1)^3}\, dx$

12. $\int \frac{1}{(3x+1)^2}\, dx$

13. $\int \frac{1}{\sqrt{x+1}}\, dx$

14. $\int \frac{1}{\sqrt{5x+1}}\, dx$

15. $\int \frac{e^{3x}}{1-e^{3x}}\, dx$

16. $\int \frac{4e^{2x}}{1+e^{2x}}\, dx$

17. $\int \frac{2x}{e^{3x^2}}\, dx$

18. $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\, dx$

19. $\int \frac{x^2}{x-1}\, dx$

20. $\int \frac{2x}{x-4}\, dx$

21. $\int x\sqrt{x^2+4}\, dx$

22. $\int \frac{t}{\sqrt{1-t^2}}\, dt$

23. $\int e^{5x}\, dx$

24. $\int te^{t^2+1}\, dt$

25. $\int \frac{e^{-x}}{e^{-x}+2}\, dx$

26. $\int \frac{e^x}{1+e^x}\, dx$

27. $\int \frac{x}{(x+1)^4}\, dx$

28. $\int \frac{x^2}{(x+1)^3}\, dx$

29. $\int \frac{x}{(3x-1)^2}\, dx$

30. $\int \frac{5x}{(x-4)^3}\, dx$

31. $\int \frac{1}{\sqrt{t}-1}\, dt$

32. $\int \frac{1}{\sqrt{x}+1}\, dx$

33. $\int \frac{2\sqrt{t}+1}{t}\, dt$

34. $\int \frac{6x+\sqrt{x}}{x}\, dx$

35. $\int \frac{x}{\sqrt{2x+1}}\, dx$

36. $\int \frac{x^2}{\sqrt{x-1}}\, dx$

37. $\int t^2\sqrt{1-t}\, dt$

38. $\int y^2\sqrt[3]{y+1}\, dy$

In Exercises 39–46, evaluate the definite integral.

39. $\int_0^4 \sqrt{2x+1}\, dx$

40. $\int_2^4 \sqrt{4x+1}\, dx$

41. $\int_0^1 3xe^{x^2}\, dx$

42. $\int_0^2 e^{-2x}\, dx$

43. $\int_0^4 \frac{x}{(x+4)^2}\, dx$

44. $\int_0^1 x(x+5)^4\, dx$

45. $\int_0^{0.5} x(1-x)^3\, dx$

46. $\int_0^{0.5} x^2(1-x)^3\, dx$

In Exercises 47–54, fin
graphs of the equatio
region and verify your

47. $y = x\sqrt{x-3},\ y$

48. $y = x\sqrt{2x+1},$

49. $y = x^2\sqrt{1-x},$

50. $y = x^2\sqrt{x+2},$

51. $y = \frac{x^2-1}{\sqrt{2x-1}},$

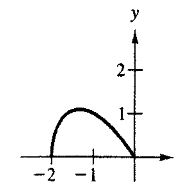
52. $y = \frac{2x-1}{\sqrt{x+3}},\ y$

53. $y = x\sqrt[3]{x+1},\ y$

54. $y = x\sqrt[3]{x-2},\ y$

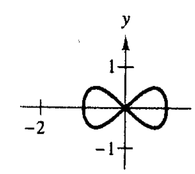
In Exercises 55–58, fin
graphs of the equatior

55. $y = -x\sqrt{x+2},$



57. $y^2 = x^2(1-x^2)$

(*Hint:* Find the ar
the region bounde
$y = x\sqrt{1-x^2}$ an
Then multiply by



In Exercises 59 and 60,
revolving the region bo
about the x-axis.

59. $y = x\sqrt{1-x^2}$

60. $y = \sqrt{x}(1-x)^2,$

In Exercises 61 and 62
function f exceeds the

61. $f(x) = \frac{1}{x+1},\ g($

62. $f(x) = x\sqrt{4x+1}$

earlier sections. You will

26. $\int \frac{e^x}{1+e^x}\,dx$

28. $\int \frac{x^2}{(x+1)^3}\,dx$

30. $\int \frac{5x}{(x-4)^3}\,dx$

32. $\int \frac{1}{\sqrt{x}+1}\,dx$

34. $\int \frac{6x+\sqrt{x}}{x}\,dx$

36. $\int \frac{x^2}{\sqrt{x-1}}\,dx$

38. $\int y^2\sqrt[3]{y+1}\cdot dy$

finite integral.

0. $\int_2^4 \sqrt{4x+1}\,dx$

2. $\int_0^2 e^{-2x}\,dx$

4. $\int_0^1 x(x+5)^4\,dx$

6. $\int_0^{0.5} x^2(1-x)^3\,dx$

In Exercises 47–54, find the area of the region bounded by the graphs of the equations. Then use a graphing utility to graph the region and verify your answer.

47. $y = x\sqrt{x-3},\ y = 0,\ x = 7$

48. $y = x\sqrt{2x+1},\ y = 0,\ x = 4$

49. $y = x^2\sqrt{1-x},\ y = 0,\ x = -3$

50. $y = x^2\sqrt{x+2},\ y = 0,\ x = 7$

51. $y = \dfrac{x^2-1}{\sqrt{2x-1}},\ y = 0,\ x = 1,\ x = 5$

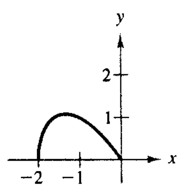
52. $y = \dfrac{2x-1}{\sqrt{x+3}},\ y = 0,\ x = \dfrac{1}{2},\ x = 6$

53. $y = x\sqrt[3]{x+1},\ y = 0,\ x = 0,\ x = 7$

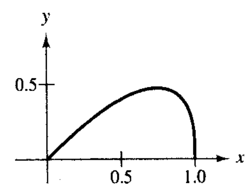
54. $y = x\sqrt[3]{x-2},\ y = 0,\ x = 2,\ x = 10$

In Exercises 55–58, find the area of the region bounded by the graphs of the equations.

55. $y = -x\sqrt{x+2},\ y = 0$

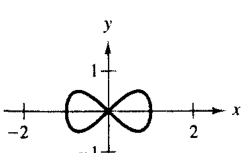


56. $y = x\sqrt[3]{1-x},\ y = 0$

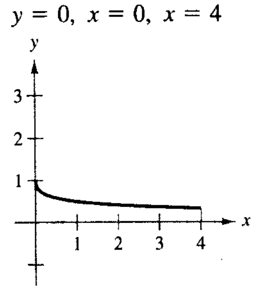


57. $y^2 = x^2(1-x^2)$

(*Hint:* Find the area of the region bounded by $y = x\sqrt{1-x^2}$ and $y = 0$. Then multiply by 4.)



58. $y = 1/(1+\sqrt{x}),$
$y = 0,\ x = 0,\ x = 4$



In Exercises 59 and 60, find the volume of the solid generated by revolving the region bounded by the graph(s) of the equation(s) about the x-axis.

59. $y = x\sqrt{1-x^2}$

60. $y = \sqrt{x}(1-x)^2,\ y = 0$

In Exercises 61 and 62, find the average amount by which the function f exceeds the function g on the interval.

61. $f(x) = \dfrac{1}{x+1},\ g(x) = \dfrac{x}{(x+1)^2},\ [0, 1]$

62. $f(x) = x\sqrt{4x+1},\ g(x) = 2\sqrt{x^3},\ [0, 2]$

63. ***Probability*** The probability of recall in an experiment modeled by

$$P(a \le x \le b) = \int_a^b \frac{15}{4}x\sqrt{1-x}\,dx$$

where x is the percent of recall (see figure).

(a) What is the probability of recalling between 40% ar 80%?

(b) What is the median percent recall? That is, for wh value of b is $P(0 \le x \le b) = 0.5$?

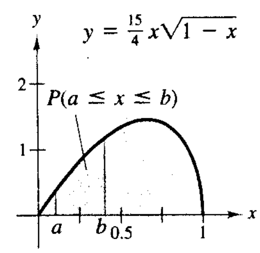


Figure for 63

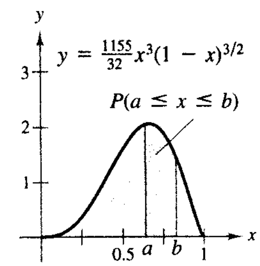


Figure for 64

64. ***Probability*** The probability of finding between a and percent iron in ore samples is modeled by

$$P(a \le x \le b) = \int_a^b \frac{1155}{32}x^3(1-x)^{3/2}\,dx$$

(see figure). Find the probabilities that a sample will co tain between (a) 0% and 25% and (b) 50% and 100% irc

65. ***Meteorology*** During a two-week period in March in small town near Lake Erie, the measurable snowfall S (inches) on the ground can be modeled by

$$S(t) = t\sqrt{14-t}, \quad 0 \le t \le 14$$

where t represents the day.

(a) Use a graphing utility to graph the function.

(b) Find the average amount of snow on the ground duri the two-week period.

(c) Find the total snowfall over the two-week period.

66. ***Revenue*** A company sells a seasonal product that ge erates a daily revenue R (in dollars per day) modeled by

$$R = 0.06t^2(365-t)^{1/2} + 1250, \quad 0 \le t \le 365$$

where t represents the day.

(a) Find the average daily revenue over a period of 1 ye

(b) Describe a product whose seasonal sales pattern rese bles the model. Explain your reasoning.

In Exercises 67 and 68, use a program similar to the Midpoint R program on page 366 with $n = 10$ to approximate the area the region bounded by the graph(s) of the equation(s).

67. $y = \sqrt[3]{x}\sqrt{4-x},\quad y = 0$

68. $y^2 = x^2(1-x^2)$

o be given by

See Figure 6.5(a).

he present value of this

$te^{-0.1t}\,dt.$

$dt = -10e^{-0.1t}$

See Figure 6.5(b).

to be given by

lation rate of 10%,

PREREQUISITE REVIEW 6.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find $f'(x)$.

1. $f(x) = \ln(x + 1)$

2. $f(x) = \ln(x^2 - 1)$

3. $f(x) = e^{x^3}$

4. $f(x) = e^{-x^2}$

5. $f(x) = x^2e^x$

6. $f(x) = xe^{-2x}$

In Exercises 7–10, find the area between the graphs of f and g.

7. $f(x) = -x^2 + 4,\ g(x) = x^2 - 4$

8. $f(x) = -x^2 + 2,\ g(x) = 1$

9. $f(x) = 4x,\ g(x) = x^2 - 5$

10. $f(x) = x^3 - 3x^2 + 2,\ g(x) = x - 1$

EXERCISES 6.2

In Exercises 1–6, use integration by parts to find the indefinite integral.

1. $\int xe^{3x}\,dx$

2. $\int xe^{-x}\,dx$

3. $\int x^2e^{-x}\,dx$

4. $\int x^2e^{2x}\,dx$

5. $\int \ln 2x\,dx$

6. $\int \ln x^2\,dx$

In Exercises 7–28, find the indefinite integral. (*Hint:* Integration by parts is not required for all the integrals.)

7. $\int e^{4x}\,dx$

8. $\int e^{-2x}\,dx$

9. $\int xe^{4x}\,dx$

10. $\int xe^{-2x}\,dx$

11. $\int xe^{x^2}\,dx$

12. $\int x^2e^{x^3}\,dx$

13. $\int x^2e^x\,dx$

14. $\int \frac{x}{e^x}\,dx$

15. $\int t\ln(t + 1)\,dt$

16. $\int x^3\ln x\,dx$

17. $\int \frac{e^{1/t}}{t^2}\,dt$

18. $\int \frac{1}{x(\ln x)^3}\,dx$

19. $\int x(\ln x)^2\,dx$

20. $\int \ln 3x\,dx$

21. $\int \frac{(\ln x)^2}{x}\,dx$

22. $\int \frac{1}{x\ln x}\,dx$

23. $\int x\sqrt{x - 1}\,dx$

24. $\int \frac{x}{\sqrt{x - 1}}\,dx$

25. $\int x(x + 1)^2\,dx$

26. $\int \frac{x}{\sqrt{2 + 3x}}\,dx$

27. $\int \frac{xe^{2x}}{(2x + 1)^2}\,dx$

28. $\int \frac{x^3e^{x^2}}{(x^2 + 1)^2}\,dx$

In Exercises 29–34, evaluate the definite integral.

29. $\int_0^1 x^2e^x\,dx$

30. $\int_0^2 \frac{x^2}{e^x}\,dx$

31. $\int_1^e x^5\ln x\,dx$

32. $\int_1^e 2x\ln x\,dx$

33. $\int_{-1}^0 \ln(x + 2)\,dx$

34. $\int_0^1 \ln(1 + 2x)\,dx$

In Exercises 35–38, find the area of the region bounded by the graphs of the equations. Then use a graphing utility to graph the region and verify your answer.

35. $y = x^3e^x,\ y = 0,\ x = 0,\ x = 2$

36. $y = (x^2 - 1)e^x,\ y = 0,\ x = -1,\ x = 1$

37. $y = x^2\ln x,\ y = 0,\ x = 1,\ x = e$

38. $y = \frac{\ln x}{x^2},\ y = 0,\ x = 1,\ x = e$

In Exercises 39–42, find the indefinite integral using each specified method. Then write a brief statement explaining which method you prefer.

39. $\int 2x\sqrt{2x-3}\,dx$

(a) By parts, letting $dv = \sqrt{2x-3}\,dx$

(b) By substitution, letting $u = \sqrt{2x-3}$

40. $\int x\sqrt{4+x}\,dx$

(a) By parts, letting $dv = \sqrt{4+x}\,dx$

(b) By substitution, letting $u = \sqrt{4+x}$

41. $\int \frac{x}{\sqrt{4+5x}}\,dx$

(a) By parts, letting $dv = \frac{1}{\sqrt{4+5x}}\,dx$

(b) By substitution, letting $u = \sqrt{4+5x}$

42. $\int x\sqrt{4-x}\,dx$

(a) By parts, letting $dv = \sqrt{4-x}\,dx$

(b) By substitution, letting $u = \sqrt{4-x}$

In Exercises 43 and 44, use integration by parts to verify the formula.

43. $\int x^n \ln x\,dx = \frac{x^{n+1}}{(n+1)^2}[-1 + (n+1)\ln x] + C,$

$n \neq -1$

44. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$

In Exercises 45–48, use the results of Exercises 43 and 44 to find the indefinite integral.

45. $\int x^2 e^{5x}\,dx$

46. $\int xe^{-3x}\,dx$

47. $\int x^{-2}\ln x\,dx$

48. $\int x^{1/2}\ln x\,dx$

In Exercises 49–52, find the area of the region bounded by the graphs of the given equations.

49. $y = xe^{-x},\ y = 0,\ x = 4$

50. $y = \frac{1}{9}xe^{-x/3},\ y = 0,\ x = 0,\ x = 3$

51. $y = x\ln x,\ y = 0,\ x = e$

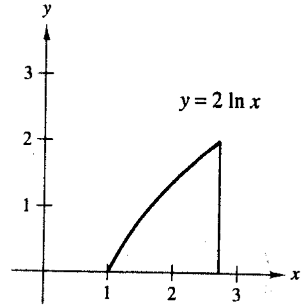
52. $y = x^{-3}\ln x,\ y = 0,\ x = e$

53. Given the region bounded by the graphs of $y = 2\ln x$, $y = 0$, and $x = e$ (see figure), find

(a) the area of the region.

(b) the volume of the solid generated by revolving the region about the x-axis.

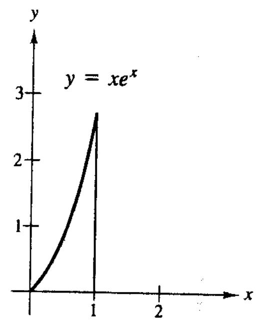
Figure for 53



54. Given the region bounded by the graphs of $y = xe^x$, $y = 0$, $x = 0$, and $x = 1$ (see figure), find

(a) the area of the region.

(b) the volume of the solid generated by revolving the region about the x-axis.



In Exercises 55–58, use a symbolic integration utility to evaluate the integral.

55. $\int_0^2 t^3 e^{-4t}\,dt$

56. $\int_1^4 \ln x(x^2+4)\,dx$

57. $\int_0^5 x^4(25-x^2)^{3/2}\,dx$

58. $\int_1^e x^9 \ln x\,dx$

59. ***Demand*** A manufacturing company forecasts that the demand x (in units per year) for its product over the next 10 years can be modeled by $x = 500(20 + te^{-0.1t})$ for $0 \le t \le 10$, where t is the time in years.

(a) Use a graphing utility to decide whether the company is forecasting an increase or a decrease in demand over the decade.

(b) According to the model, what is the total demand over the next 10 years?

(c) Find the average annual demand during the 10-year period.

60. ***Capital Campaign*** The board of trustees of a college is planning a five-year capital gifts campaign to raise money for the college. The goal is to have an annual gift income I that is modeled by $I = 2000(375 + 68te^{-0.2t})$ for $0 \le t \le 5$, where t is the time in years.

(a) Use a graphi
trustees expe
over the five-

(b) Find the exp
period.

(c) Determine th
year period. (
when $t = 3$.

61. ***Learning Theo***
to memorize, me

$M = 1 + 1.6t$

where t is the chi
of this model bet

(a) the child's fir

(b) the child's thi

62. ***Revenue*** A c
revenue R (in do
product can be m

$R = 410.5t^2e^-$

where t is the tim

(a) Find the aver
which is give

(b) Find the aver
ter, which is

(c) Find the total

Present Value In E
the income c (measu
annual inflation rate r.

63. $c = 5000,\ r = 5$

64. $c = 450,\ r = 4\%$

65. $c = 150{,}000 + 2$

66. $c = 30{,}000 + 50$

67. $c = 1000 + 50e^{t}$

68. $c = 5000 + 25te$

69. ***Present Value***
the next 4 years t

$c = 150{,}000 +$

(a) Find the act
4 years.

(b) Assuming an
present value

(a) Use a graphing utility to decide whether the board of trustees expects the gift income to increase or decrease over the five-year period.

(b) Find the expected total gift income over the five-year period.

(c) Determine the average annual gift income over the five-year period. Compare the result with the income given when $t = 3$.

61. ***Learning Theory*** A model for the ability M of a child to memorize, measured on a scale from 0 to 10, is

$$M = 1 + 1.6t \ln t, \qquad 0 < t \le 4$$

where t is the child's age in years. Find the average value of this model between

(a) the child's first and second birthdays.

(b) the child's third and fourth birthdays.

62. ***Revenue*** A company sells a seasonal product. The revenue R (in dollars per year) generated by sales of the product can be modeled by

$$R = 410.5t^2e^{-t/30} + 25{,}000, \qquad 0 \le t \le 365$$

where t is the time in days.

(a) Find the average daily receipts during the first quarter, which is given by $0 \le t \le 90$.

(b) Find the average daily receipts during the fourth quarter, which is given by $274 \le t \le 365$.

(c) Find the total daily receipts during the year.

Present Value In Exercises 63–68, find the present value of the income c (measured in dollars) over t_1 years at the given annual inflation rate r.

63. $c = 5000,\ r = 5\%,\ t_1 = 4$ years

64. $c = 450,\ r = 4\%,\ t_1 = 10$ years

65. $c = 150{,}000 + 2500t,\ r = 4\%,\ t_1 = 10$ years

66. $c = 30{,}000 + 500t,\ r = 7\%,\ t_1 = 6$ years

67. $c = 1000 + 50e^{t/2},\ r = 6\%,\ t_1 = 4$ years

68. $c = 5000 + 25te^{t/10},\ r = 6\%,\ t_1 = 10$ years

69. ***Present Value*** A company expects its income c during the next 4 years to be modeled by

$$c = 150{,}000 + 75{,}000t.$$

(a) Find the actual income for the business over the 4 years.

(b) Assuming an annual inflation rate of 4%, what is the present value of this income?

70. ***Present Value*** A professional athlete signs a three-year contract in which the earnings can be modeled by

$$c = 300{,}000 + 125{,}000t.$$

(a) Find the actual value of the athlete's contract.

(b) Assuming an annual inflation rate of 5%, what is the present value of the contract?

Future Value In Exercises 71 and 72, find the future value of the income (in dollars) given by $f(t)$ over t_1 years at the annual interest rate of r. If the function f represents a continuous investment over a period of t_1 years at an annual interest rate of r (compounded continuously), then the future value of the investment is given by

$$\text{Future value} = e^{rt_1}\int_0^{t_1} f(t)e^{-rt}\,dt.$$

71. $f(t) = 3000,\ r = 8\%,\ t_1 = 10$ years

72. $f(t) = 3000e^{0.05t},\ r = 10\%,\ t_1 = 5$ years

73. ***Finance: Future Value*** Use the equation from Exercises 71 and 72 to calculate the following. *(Source: Adapted from Garman/Forgue,* Personal Finance, *Fifth Edition)*

(a) The future value of \$1200 saved each year for 10 years earning 7% interest.

(b) A person who wishes to invest \$1200 each year finds one investment choice that is expected to pay 9% interest per year and another, riskier choice that may pay 10% interest per year. What is the difference in return (future value) if the investment is made for 15 years?

74. ***Consumer Awareness*** In 2004, the total cost to attend Pennsylvania State University for 1 year was estimated to be \$19,843. If your grandparents had continuously invested in a college fund according to the model

$$f(t) = 400t$$

for 18 years, at an annual interest rate of 10%, would the fund have grown enough to allow you to cover 4 years of expenses at Pennsylvania State University? *(Source: Pennsylvania State University)*

75. Use a program similar to the Midpoint Rule program on page 366 with $n = 10$ to approximate

$$\int_1^4 \frac{4}{\sqrt{x} + \sqrt[3]{x}}\,dx.$$

76. Use a program similar to the Midpoint Rule program on page 366 with $n = 12$ to approximate the volume of the solid generated by revolving the region bounded by the graphs of

$$y = \frac{10}{\sqrt{xe^x}},\ y = 0,\ x = 1,\ \text{and}\ x = 4$$

about the x-axis.