o be given by

See Figure 6.5(a).

he present value of this

$te^{-0.1t}\,dt$.

$dt = -10e^{-0.1t}$

See Figure 6.5(b).

to be given by
lation rate of 10%,

PREREQUISITE REVIEW 6.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find $f'(x)$.

1. $f(x) = \ln(x + 1)$

2. $f(x) = \ln(x^2 - 1)$

3. $f(x) = e^{x^3}$

4. $f(x) = e^{-x^2}$

5. $f(x) = x^2e^x$

6. $f(x) = xe^{-2x}$

In Exercises 7–10, find the area between the graphs of f and g.

7. $f(x) = -x^2 + 4,\ g(x) = x^2 - 4$

8. $f(x) = -x^2 + 2,\ g(x) = 1$

9. $f(x) = 4x,\ g(x) = x^2 - 5$

10. $f(x) = x^3 - 3x^2 + 2,\ g(x) = x - 1$

EXERCISES 6.2

In Exercises 1–6, use integration by parts to find the indefinite integral.

1. $\displaystyle\int xe^{3x}\,dx$

2. $\displaystyle\int xe^{-x}\,dx$

3. $\displaystyle\int x^2e^{-x}\,dx$

4. $\displaystyle\int x^2e^{2x}\,dx$

5. $\displaystyle\int \ln 2x\,dx$

6. $\displaystyle\int \ln x^2\,dx$

In Exercises 7–28, find the indefinite integral. (*Hint:* Integration by parts is not required for all the integrals.)

7. $\displaystyle\int e^{4x}\,dx$

8. $\displaystyle\int e^{-2x}\,dx$

9. $\displaystyle\int xe^{4x}\,dx$

10. $\displaystyle\int xe^{-2x}\,dx$

11. $\displaystyle\int xe^{x^2}\,dx$

12. $\displaystyle\int x^2e^{x^3}\,dx$

13. $\displaystyle\int x^2e^{x}\,dx$

14. $\displaystyle\int \frac{x}{e^x}\,dx$

15. $\displaystyle\int t\ln(t + 1)\,dt$

16. $\displaystyle\int x^3\ln x\,dx$

17. $\displaystyle\int \frac{e^{1/t}}{t^2}\,dt$

18. $\displaystyle\int \frac{1}{x(\ln x)^3}\,dx$

19. $\displaystyle\int x(\ln x)^2\,dx$

20. $\displaystyle\int \ln 3x\,dx$

21. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

22. $\displaystyle\int \frac{1}{x\ln x}\,dx$

23. $\displaystyle\int x\sqrt{x - 1}\,dx$

24. $\displaystyle\int \frac{x}{\sqrt{x - 1}}\,dx$

25. $\displaystyle\int x(x + 1)^2\,dx$

26. $\displaystyle\int \frac{x}{\sqrt{2 + 3x}}\,dx$

27. $\displaystyle\int \frac{xe^{2x}}{(2x + 1)^2}\,dx$

28. $\displaystyle\int \frac{x^3e^{x^2}}{(x^2 + 1)^2}\,dx$

In Exercises 29–34, evaluate the definite integral.

29. $\displaystyle\int_0^1 x^2e^x\,dx$

30. $\displaystyle\int_0^2 \frac{x^2}{e^x}\,dx$

31. $\displaystyle\int_1^e x^5\ln x\,dx$

32. $\displaystyle\int_1^e 2x\ln x\,dx$

33. $\displaystyle\int_{-1}^0 \ln(x + 2)\,dx$

34. $\displaystyle\int_0^1 \ln(1 + 2x)\,dx$

In Exercises 35–38, find the area of the region bounded by the graphs of the equations. Then use a graphing utility to graph the region and verify your answer.

35. $y = x^3e^x,\ y = 0,\ x = 0,\ x = 2$

36. $y = (x^2 - 1)e^x,\ y = 0,\ x = -1,\ x = 1$

37. $y = x^2\ln x,\ y = 0,\ x = 1,\ x = e$

38. $y = \dfrac{\ln x}{x^2},\ y = 0,\ x = 1,\ x = e$

In Exercises 39–42, find the indefinite integral using each specified method. Then write a brief statement explaining which method you prefer.

39. $\int 2x\sqrt{2x-3}\,dx$

(a) By parts, letting $dv = \sqrt{2x-3}\,dx$

(b) By substitution, letting $u = \sqrt{2x-3}$

40. $\int x\sqrt{4+x}\,dx$

(a) By parts, letting $dv = \sqrt{4+x}\,dx$

(b) By substitution, letting $u = \sqrt{4+x}$

41. $\int \frac{x}{\sqrt{4+5x}}\,dx$

(a) By parts, letting $dv = \frac{1}{\sqrt{4+5x}}\,dx$

(b) By substitution, letting $u = \sqrt{4+5x}$

42. $\int x\sqrt{4-x}\,dx$

(a) By parts, letting $dv = \sqrt{4-x}\,dx$

(b) By substitution, letting $u = \sqrt{4-x}$

In Exercises 43 and 44, use integration by parts to verify the formula.

43. $\int x^n \ln x\,dx = \frac{x^{n+1}}{(n+1)^2}[-1 + (n+1)\ln x] + C,$

$n \neq -1$

44. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$

In Exercises 45–48, use the results of Exercises 43 and 44 to find the indefinite integral.

45. $\int x^2 e^{5x}\,dx$

46. $\int xe^{-3x}\,dx$

47. $\int x^{-2}\ln x\,dx$

48. $\int x^{1/2}\ln x\,dx$

In Exercises 49–52, find the area of the region bounded by the graphs of the given equations.

49. $y = xe^{-x},\ y = 0,\ x = 4$

50. $y = \frac{1}{9}xe^{-x/3},\ y = 0,\ x = 0,\ x = 3$

51. $y = x\ln x,\ y = 0,\ x = e$

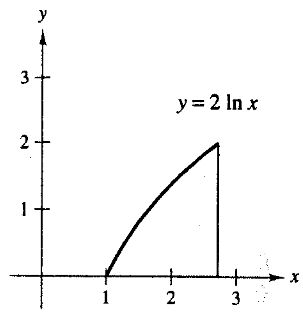
52. $y = x^{-3}\ln x,\ y = 0,\ x = e$

53. Given the region bounded by the graphs of $y = 2\ln x$, $y = 0$, and $x = e$ (see figure), find

(a) the area of the region.

(b) the volume of the solid generated by revolving the region about the x-axis.

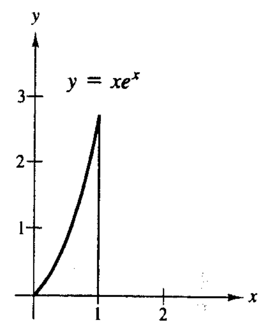
Figure for 53



54. Given the region bounded by the graphs of $y = xe^x$, $y = 0$, $x = 0$, and $x = 1$ (see figure), find

(a) the area of the region.

(b) the volume of the solid generated by revolving the region about the x-axis.



In Exercises 55–58, use a symbolic integration utility to evaluate the integral.

55. $\int_0^2 t^3 e^{-4t}\,dt$

56. $\int_1^4 \ln x(x^2+4)\,dx$

57. $\int_0^5 x^4(25-x^2)^{3/2}\,dx$

58. $\int_1^e x^9 \ln x\,dx$

59. ***Demand*** A manufacturing company forecasts that the demand x (in units per year) for its product over the next 10 years can be modeled by $x = 500(20 + te^{-0.1t})$ for $0 \leq t \leq 10$, where t is the time in years.

(a) Use a graphing utility to decide whether the company is forecasting an increase or a decrease in demand over the decade.

(b) According to the model, what is the total demand over the next 10 years?

(c) Find the average annual demand during the 10-year period.

60. ***Capital Campaign*** The board of trustees of a college is planning a five-year capital gifts campaign to raise money for the college. The goal is to have an annual gift income I that is modeled by $I = 2000(375 + 68te^{-0.2t})$ for $0 \leq t \leq 5$, where t is the time in years.

(a) Use a graphi
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over the five-

(b) Find the exp
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(c) Determine th
year period. (
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61. ***Learning Theo***
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62. ***Revenue*** A c
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(b) Find the aver
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(c) Find the total

Present Value In E
the income c (measu
annual inflation rate r.

63. $c = 5000,\ r = 5$

64. $c = 450,\ r = 4\%$

65. $c = 150{,}000 + 2$

66. $c = 30{,}000 + 50$

67. $c = 1000 + 50e^{t/}$

68. $c = 5000 + 25te$

69. ***Present Value***
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$c = 150{,}000 +$

(a) Find the act
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(b) Assuming an
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$^{-0.2t}$) for $0 \le t \le 5$,

(a) Use a graphing utility to decide whether the board of trustees expects the gift income to increase or decrease over the five-year period.

(b) Find the expected total gift income over the five-year period.

(c) Determine the average annual gift income over the five-year period. Compare the result with the income given when $t = 3$.

61. ***Learning Theory*** A model for the ability M of a child to memorize, measured on a scale from 0 to 10, is

$$M = 1 + 1.6t \ln t, \qquad 0 < t \le 4$$

where t is the child's age in years. Find the average value of this model between

(a) the child's first and second birthdays.

(b) the child's third and fourth birthdays.

62. ***Revenue*** A company sells a seasonal product. The revenue R (in dollars per year) generated by sales of the product can be modeled by

$$R = 410.5t^2e^{-t/30} + 25{,}000, \qquad 0 \le t \le 365$$

where t is the time in days.

(a) Find the average daily receipts during the first quarter, which is given by $0 \le t \le 90$.

(b) Find the average daily receipts during the fourth quarter, which is given by $274 \le t \le 365$.

(c) Find the total daily receipts during the year.

Present Value In Exercises 63–68, find the present value of the income c (measured in dollars) over t_1 years at the given annual inflation rate r.

63. $c = 5000$, $r = 5\%$, $t_1 = 4$ years

64. $c = 450$, $r = 4\%$, $t_1 = 10$ years

65. $c = 150{,}000 + 2500t$, $r = 4\%$, $t_1 = 10$ years

66. $c = 30{,}000 + 500t$, $r = 7\%$, $t_1 = 6$ years

67. $c = 1000 + 50e^{t/2}$, $r = 6\%$, $t_1 = 4$ years

68. $c = 5000 + 25te^{t/10}$, $r = 6\%$, $t_1 = 10$ years

69. ***Present Value*** A company expects its income c during the next 4 years to be modeled by

$$c = 150{,}000 + 75{,}000t.$$

(a) Find the actual income for the business over the 4 years.

(b) Assuming an annual inflation rate of 4%, what is the present value of this income?

70. ***Present Value*** A professional athlete signs a three-year contract in which the earnings can be modeled by

$$c = 300{,}000 + 125{,}000t.$$

(a) Find the actual value of the athlete's contract.

(b) Assuming an annual inflation rate of 5%, what is the present value of the contract?

Future Value In Exercises 71 and 72, find the future value of the income (in dollars) given by $f(t)$ over t_1 years at the annual interest rate of r. If the function f represents a continuous investment over a period of t_1 years at an annual interest rate of r (compounded continuously), then the future value of the investment is given by

$$\text{Future value} = e^{rt_1}\int_0^{t_1} f(t)e^{-rt}\, dt.$$

71. $f(t) = 3000$, $r = 8\%$, $t_1 = 10$ years

72. $f(t) = 3000e^{0.05t}$, $r = 10\%$, $t_1 = 5$ years

73. ***Finance: Future Value*** Use the equation from Exercises 71 and 72 to calculate the following. *(Source: Adapted from Garman/Forgue,* Personal Finance, *Fifth Edition)*

(a) The future value of \$1200 saved each year for 10 years earning 7% interest.

(b) A person who wishes to invest \$1200 each year finds one investment choice that is expected to pay 9% interest per year and another, riskier choice that may pay 10% interest per year. What is the difference in return (future value) if the investment is made for 15 years?

74. ***Consumer Awareness*** In 2004, the total cost to attend Pennsylvania State University for 1 year was estimated to be \$19,843. If your grandparents had continuously invested in a college fund according to the model

$$f(t) = 400t$$

for 18 years, at an annual interest rate of 10%, would the fund have grown enough to allow you to cover 4 years of expenses at Pennsylvania State University? *(Source: Pennsylvania State University)*

75. Use a program similar to the Midpoint Rule program on page 366 with $n = 10$ to approximate

$$\int_1^4 \frac{4}{\sqrt{x} + \sqrt[3]{x}}\, dx.$$

76. Use a program similar to the Midpoint Rule program on page 366 with $n = 12$ to approximate the volume of the solid generated by revolving the region bounded by the graphs of

$$y = \frac{10}{\sqrt{x}e^x}, \quad y = 0, \quad x = 1, \text{ and } x = 4$$

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PREREQUISITE REVIEW 6.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, factor the expression.

1. $x^2 - 16$

2. $x^2 - 25$

3. $x^2 - x - 12$

4. $x^2 + x - 6$

5. $x^3 - x^2 - 2x$

6. $x^3 - 4x^2 + 4x$

7. $x^3 - 4x^2 + 5x - 2$

8. $x^3 - 5x^2 + 7x - 3$

In Exercises 9–14, rewrite the improper rational expression as the sum of a proper rational expression and a polynomial.

9. $\dfrac{x^2 - 2x + 1}{x - 2}$

10. $\dfrac{2x^2 - 4x + 1}{x - 1}$

11. $\dfrac{x^3 - 3x^2 + 2}{x - 2}$

12. $\dfrac{x^3 + 2x - 1}{x + 1}$

13. $\dfrac{x^3 + 4x^2 + 5x + 2}{x^2 - 1}$

14. $\dfrac{x^3 + 3x^2 - 4}{x^2 - 1}$

EXERCISES 6.3

In Exercises 1–12, write the partial fraction decomposition for the expression.

1. $\dfrac{2(x + 20)}{x^2 - 25}$

2. $\dfrac{3x + 11}{x^2 - 2x - 3}$

3. $\dfrac{8x + 3}{x^2 - 3x}$

4. $\dfrac{10x + 3}{x^2 + x}$

5. $\dfrac{4x - 13}{x^2 - 3x - 10}$

6. $\dfrac{7x + 5}{6(2x^2 + 3x + 1)}$

7. $\dfrac{3x^2 - 2x - 5}{x^3 + x^2}$

8. $\dfrac{3x^2 - x + 1}{x(x + 1)^2}$

9. $\dfrac{x + 1}{3(x - 2)^2}$

10. $\dfrac{3x - 4}{(x - 5)^2}$

11. $\dfrac{8x^2 + 15x + 9}{(x + 1)^3}$

12. $\dfrac{6x^2 - 5x}{(x + 2)^3}$

In Exercises 13–32, find the indefinite integral.

13. $\displaystyle\int \frac{1}{x^2 - 1}\,dx$

14. $\displaystyle\int \frac{9}{x^2 - 9}\,dx$

15. $\displaystyle\int \frac{-2}{x^2 - 16}\,dx$

16. $\displaystyle\int \frac{-4}{x^2 - 4}\,dx$

17. $\displaystyle\int \frac{1}{3x^2 - x}\,dx$

18. $\displaystyle\int \frac{3}{x^2 - 3x}\,dx$

19. $\displaystyle\int \frac{1}{2x^2 + x}\,dx$

20. $\displaystyle\int \frac{5}{x^2 + x - 6}\,dx$

21. $\displaystyle\int \frac{3}{x^2 + x - 2}\,dx$

22. $\displaystyle\int \frac{1}{4x^2 - 9}\,dx$

23. $\displaystyle\int \frac{5 - x}{2x^2 + x - 1}\,dx$

24. $\displaystyle\int \frac{x + 1}{x^2 + 4x + 3}\,dx$

25. $\displaystyle\int \frac{x^2 + 12x + 12}{x^3 - 4x}\,dx$

26. $\displaystyle\int \frac{3x^2 - 7x - 2}{x^3 - x}\,dx$

27. $\displaystyle\int \frac{x + 2}{x^2 - 4x}\,dx$

28. $\displaystyle\int \frac{4x^2 + 2x - 1}{x^3 + x^2}\,dx$

29. $\displaystyle\int \frac{4 - 3x}{(x - 1)^2}\,dx$

30. $\displaystyle\int \frac{x^4}{(x - 1)^3}\,dx$

31. $\displaystyle\int \frac{3x^2 + 3x + 1}{x(x^2 + 2x + 1)}\,dx$

32. $\displaystyle\int \frac{3x}{x^2 - 6x + 9}\,dx$

In Exercises 33–40, evaluate the definite integral.

33. $\displaystyle\int_4^5 \frac{1}{9 - x^2}\,dx$

34. $\displaystyle\int_0^1 \frac{3}{2x^2 + 5x + 2}\,dx$

35. $\displaystyle\int_1^5 \frac{x - 1}{x^2(x + 1)}\,dx$

36. $\displaystyle\int_0^1 \frac{x^2 - x}{x^2 + x + 1}\,dx$

37. $\displaystyle\int_0^1 \frac{x^3}{x^2-2}\,dx$

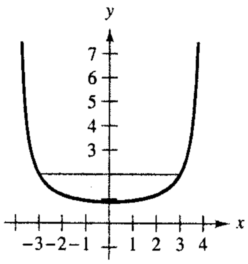
38. $\displaystyle\int_0^1 \frac{x^3-1}{x^2-4}\,dx$

39. $\displaystyle\int_1^2 \frac{x^3-4x^2-3x+3}{x^2-3x}\,dx$

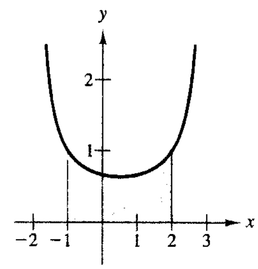
40. $\displaystyle\int_2^4 \frac{x^4-4}{x^2-1}\,dx$

In Exercises 41–44, find the area of the shaded region.

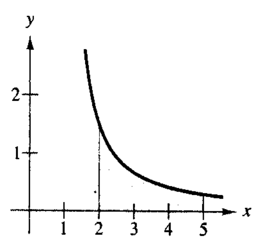
41. $y = \dfrac{14}{16-x^2}$



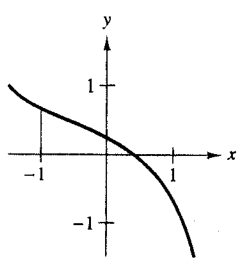
42. $y = \dfrac{-4}{x^2-x-6}$



43. $y = \dfrac{x+1}{x^2-x}$



44. $y = \dfrac{x^2+2x-1}{x^2-4}$



In Exercises 45–48, write the partial fraction decomposition for the rational expression. Check your result algebraically. Then assign a value to the constant a and use a graphing utility to check the result graphically.

45. $\dfrac{1}{a^2-x^2}$

46. $\dfrac{1}{x(x+a)}$

47. $\dfrac{1}{x(a-x)}$

48. $\dfrac{1}{(x+1)(a-x)}$

In Exercises 49–52, use a graphing utility to graph the function. Then find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the x-axis by using the integration capabilities of a graphing utility and by integrating by hand using partial fraction decomposition.

49. $y = \dfrac{10}{x(x+10)},\ y = 0,\ x = 1,\ x = 5$

50. $y = \dfrac{-4}{(x+1)(x-4)},\ y = 0,\ x = 0,\ x = 3$

51. $y = \dfrac{2}{x^2-4},\ x = 1,\ x = -1,\ y = 0$

52. $y = \dfrac{25x}{x^2+x-6},\ x = -2,\ x = 0,\ y = 0$

53. ***Biology*** A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a capacity of 1000 animals and that the herd will grow according to a logistic growth model. That is, the size y of the herd will follow the equation

$$\int \frac{1}{y(1000-y)}\,dy = \int k\,dt$$

where t is measured in years. Find this logistic curve. (To solve for the constant of integration C and the proportionality constant k, assume $y = 100$ when $t = 0$ and $y = 134$ when $t = 2$.) Use a graphing utility to graph your solution.

54. ***Health: Epidemic*** A single infected individual enters a community of 500 individuals susceptible to the disease. The disease spreads at a rate proportional to the product of the total number infected and the number of susceptible individuals not yet infected. A model for the time it takes for the disease to spread to x individuals is

$$t = 5010\int \frac{1}{(x+1)(500-x)}\,dx$$

where t is the time in hours.

(a) Find the time it takes for 75% of the population to become infected (when $t = 0$, $x = 1$).

(b) Find the number of people infected after 100 hours.

55. ***Marketing*** After test-marketing a new menu item, a fast-food restaurant predicts that sales of the new item will grow according to the model

$$\frac{dS}{dt} = \frac{2t}{(t+4)^2}$$

where t is the time in weeks and S is the sales (in thousands of dollars). Find the sales of the menu item at 10 weeks.

56. ***Biology*** One gram of a bacterial culture is present at time $t = 0$, and 10 grams is the upper limit of the culture's weight. The time required for the culture to grow to y grams is modeled by

$$kt = \int \frac{1}{y(10-y)}\,dy$$

where y is the weight of the culture (in grams) and t is the time in hours.

(a) Verify tha
by

$y = \frac{}{1}$

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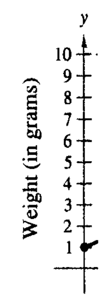
(b) Use the gı



57. ***Revenue*** T
for Symantec
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$R = \frac{410t^2}{}$

where $t = 5$
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58. ***Medicine***
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history of spr

$\frac{dN}{dt} = \frac{1}{(1-}$

where N is th

(a) Find the
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(b) If nothin
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59. ***Biology*** A
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increase at a

$\frac{dN}{dt} = \frac{1}{(1-}$

where N is th

(a) Use the fa
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(b) Find the li
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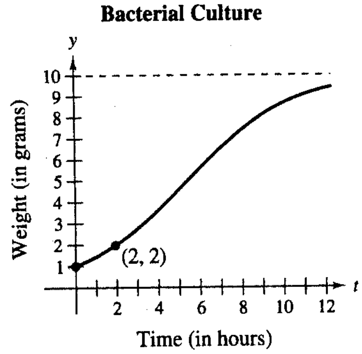
(a) Verify that the weight of the culture at time t is modeled by

$$y = \frac{10}{1 + 9e^{-10kt}}.$$

Use the fact that $y = 1$ when $t = 0$.

(b) Use the graph to determine the constant k.

Bacterial Culture



57. ***Revenue*** The revenue R (in millions of dollars per year) for Symantec Corporation from 1995 through 2003 can be modeled by

$$R = \frac{410t^2 + 28{,}490t + 28{,}080}{-6t^2 + 94t + 100}$$

where $t = 5$ corrresponds to 1995. Find the total revenue from 1995 through 2003. Then find the average revenue during this time period. *(Source: Symantec Corporation)*

58. ***Medicine*** On a college campus, 50 students return from semester break with a contagious flu virus. The virus has a history of spreading at a rate of

$$\frac{dN}{dt} = \frac{100e^{-0.1t}}{(1 + 4e^{-0.1t})^2}$$

where N is the number of students infected after t days.

(a) Find the model giving the number of students infected with the virus in terms of the number of days since returning from semester break.

(b) If nothing is done to stop the virus from spreading, will the virus spread to infect half the student population of 1000 students? Explain your answer.

59. ***Biology*** A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes the population of the species will increase at a rate of

$$\frac{dN}{dt} = \frac{125e^{-0.125t}}{(1 + 9e^{-0.125t})^2}$$

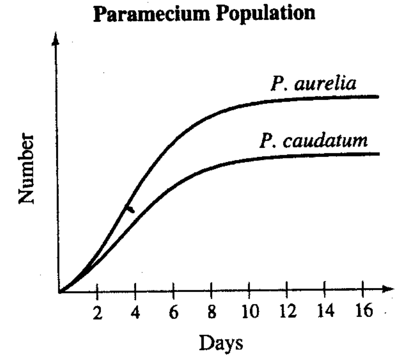
where N is the population and t is the time in months.

(a) Use the fact that $N = 100$ when $t = 0$ to find the population after 2 years.

(b) Find the limiting size of the population as time increases without bound.

60. ***Biology: Population Growth*** The graph shows the logistic growth curves for two species of the single-celled *Paramecium* in a laboratory culture. During which time intervals is the rate of growth of each species increasing? During which time intervals is the rate of growth of each species decreasing? Which species has a higher limiting population under these conditions? *(Source: Adapted from Levine/Miller,* Biology: Discovering Life, *Second Edition)*

Paramecium Population



BUSINESS CAPSULE

Courtesy of Susie Wang/Aqua Dessa

While a math communications major at the University of California at Berkeley, Susie Wang began researching the idea of selling natural skin-care products. She used $10,000 to start her company, Aqua Dessa, and uses word-of-mouth as an advertising tactic. Aqua Dessa products are used and sold at spas and exclusive cosmetics counters throughout the United States.

61. ***Research Project*** Use your school's library, the Internet, or some other reference source to research the opportunity cost of attending graduate school for 2 years to receive a Masters of Business Administration (MBA) degree rather than working for 2 years with a bachelor's degree. Write a short paper describing these costs.

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In the table of integrals below and on the next two pages, the formulas have been grouped into eight different types according to the form of the integrand.

Forms involving u^n

Forms involving $a + bu$

Forms involving $\sqrt{a + bu}$

Forms involving $\sqrt{u^2 \pm a^2}$

Forms involving $u^2 - a^2$

Forms involving $\sqrt{a^2 - u^2}$

Forms involving e^u

Forms involving $\ln u$

Table of Integrals

Forms involving u^n

1. $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

Forms involving $a + bu$

3. $\displaystyle\int \frac{u}{a+bu}\,du = \frac{1}{b^2}\left(bu - a\ln|a+bu|\right) + C$

4. $\displaystyle\int \frac{u}{(a+bu)^2}\,du = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$

5. $\displaystyle\int \frac{u}{(a+bu)^n}\,du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} + \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C, \quad n \neq 1, 2$

6. $\displaystyle\int \frac{u^2}{a+bu}\,du = \frac{1}{b^3}\left[-\frac{bu}{2}(2a - bu) + a^2\ln|a+bu|\right] + C$

7. $\displaystyle\int \frac{u^2}{(a+bu)^2}\,du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

8. $\displaystyle\int \frac{u^2}{(a+bu)^3}\,du = \frac{1}{b^3}\left[\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right] + C$

9. $\displaystyle\int \frac{u^2}{(a+bu)^n}\,du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C, \ n \neq 1, 2, 3$

10. $\displaystyle\int \frac{1}{u(a+bu)}\,du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

11. $\displaystyle\int \frac{1}{u(a+bu)^2}\,du = \frac{1}{a}\left(\frac{1}{a+bu} + \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

12. $\displaystyle\int \frac{1}{u^2(a+bu)}\,du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

13. $\displaystyle\int \frac{1}{u^2(a+bu)^2}\,du = -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} + \frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right] + C$

Table of Integrals (*continued*)

Forms involving $\sqrt{a + bu}$

14. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$

15. $\displaystyle\int \frac{1}{u\sqrt{a+bu}}\,du = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \quad a > 0$

16. $\displaystyle\int \frac{1}{u^n\sqrt{a+bu}}\,du = \frac{-1}{a(n-1)}\left[\frac{\sqrt{a+bu}}{u^{n-1}} + \frac{(2n-3)b}{2}\int\frac{1}{u^{n-1}\sqrt{a+bu}}\,du\right], \quad n \neq 1$

17. $\displaystyle\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int\frac{1}{u\sqrt{a+bu}}\,du$

18. $\displaystyle\int \frac{\sqrt{a+bu}}{u^n}\,du = \frac{-1}{a(n-1)}\left[\frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2}\int\frac{\sqrt{a+bu}}{u^{n-1}}\,du\right], \quad n \neq 1$

19. $\displaystyle\int \frac{u}{\sqrt{a+bu}}\,du = -\frac{2(2a-bu)}{3b^2}\sqrt{a+bu} + C$

20. $\displaystyle\int \frac{u^n}{\sqrt{a+bu}}\,du = \frac{2}{(2n+1)b}\left(u^n\sqrt{a+bu} - na\int\frac{u^{n-1}}{\sqrt{a+bu}}\,du\right)$

Forms involving $\sqrt{u^2 \pm a^2}, \quad a > 0$

21. $\displaystyle\int \sqrt{u^2 \pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \pm a^2\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

22. $\displaystyle\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{1}{8}\left[u(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - a^4\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right] + C$

23. $\displaystyle\int \frac{\sqrt{u^2 + a^2}}{u}\,du = \sqrt{u^2 + a^2} - a\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

24. $\displaystyle\int \frac{\sqrt{u^2 \pm a^2}}{u^2}\,du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

25. $\displaystyle\int \frac{1}{\sqrt{u^2 \pm a^2}}\,du = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

26. $\displaystyle\int \frac{1}{u\sqrt{u^2 + a^2}}\,du = -\frac{1}{a}\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

27. $\displaystyle\int \frac{u^2}{\sqrt{u^2 \pm a^2}}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

28. $\displaystyle\int \frac{1}{u^2\sqrt{u^2 \pm a^2}}\,du = \mp\frac{\sqrt{u^2 \pm a^2}}{a^2u} + C$

Table of Integ

Forms involvin

29. $\int \frac{1}{u^2 - a^2}$

30. $\int \frac{1}{(u^2 - a}$

Forms involvin

31. $\int \frac{\sqrt{a^2 -}}{u}$

32. $\int \frac{1}{u\sqrt{a^2 -}}$

33. $\int \frac{1}{u^2\sqrt{a^2}}$

Forms involvin

34. $\int e^u\,du =$

35. $\int ue^u\,du =$

36. $\int u^ne^u\,du$

37. $\int \frac{1}{1 + e^u}$

38. $\int \frac{1}{1 + e^{nu}}$

Forms involvin

39. $\int \ln u\,du =$

40. $\int u\ln u\,du$

41. $\int u^n\ln u\,d$

42. $\int (\ln u)^2\,d$

43. $\int (\ln u)^n\,d$

Table of Integrals (*continued*)

Forms involving $u^2 - a^2$, $a > 0$

29. $\displaystyle\int \frac{1}{u^2 - a^2}\,du = -\int \frac{1}{a^2 - u^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

30. $\displaystyle\int \frac{1}{(u^2 - a^2)^n}\,du = \frac{-1}{2a^2(n-1)}\left[\frac{u}{(u^2 - a^2)^{n-1}} + (2n-3)\int \frac{1}{(u^2 - a^2)^{n-1}}\,du\right], \quad n \neq 1$

Forms involving $\sqrt{a^2 - u^2}$, $a > 0$

31. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}\,du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

32. $\displaystyle\int \frac{1}{u\sqrt{a^2 - u^2}}\,du = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

33. $\displaystyle\int \frac{1}{u^2\sqrt{a^2 - u^2}}\,du = \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$

Forms involving e^u

34. $\displaystyle\int e^u\,du = e^u + C$

35. $\displaystyle\int ue^u\,du = (u-1)e^u + C$

36. $\displaystyle\int u^n e^u\,du = u^n e^u - n\int u^{n-1}e^u\,du$

37. $\displaystyle\int \frac{1}{1 + e^u}\,du = u - \ln(1 + e^u) + C$

38. $\displaystyle\int \frac{1}{1 + e^{nu}}\,du = u - \frac{1}{n}\ln(1 + e^{nu}) + C$

Forms involving $\ln u$

39. $\displaystyle\int \ln u\,du = u(-1 + \ln u) + C$

40. $\displaystyle\int u\ln u\,du = \frac{u^2}{4}(-1 + 2\ln u) + C$

41. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[-1 + (n+1)\ln u] + C, \quad n \neq -1$

42. $\displaystyle\int (\ln u)^2\,du = u[2 - 2\ln u + (\ln u)^2] + C$

43. $\displaystyle\int (\ln u)^n\,du = u(\ln u)^n - n\int (\ln u)^{n-1}\,du$

PREREQUISITE REVIEW 6.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, expand the expression.

1. $(x + 4)^2$

2. $(x - 1)^2$

3. $\left(x + \frac{1}{2}\right)^2$

4. $\left(x - \frac{1}{3}\right)^2$

In Exercises 5–8, write the partial fraction decomposition for the expression.

5. $\dfrac{4}{x(x + 2)}$

6. $\dfrac{3}{x(x - 4)}$

7. $\dfrac{x + 4}{x^2(x - 2)}$

8. $\dfrac{3x^2 + 4x - 8}{x(x - 2)(x + 1)}$

In Exercises 9 and 10, use integration by parts to find the indefinite integral.

9. $\int 2xe^x \, dx$

10. $\int 3x^2 \ln x \, dx$

EXERCISES 6.4

In Exercises 1–8, use the indicated formula from the table of integrals in this section to find the indefinite integral.

1. $\int \dfrac{x}{(2 + 3x)^2} \, dx$, Formula 4

2. $\int \dfrac{1}{x(2 + 3x)^2} \, dx$, Formula 11

3. $\int \dfrac{x}{\sqrt{2 + 3x}} \, dx$, Formula 19

4. $\int \dfrac{4}{x^2 - 9} \, dx$, Formula 29

5. $\int \dfrac{2x}{\sqrt{x^4 - 9}} \, dx$, Formula 25

6. $\int x^2\sqrt{x^2 + 9} \, dx$, Formula 22

7. $\int x^3 e^{x^2} \, dx$, Formula 35

8. $\int \dfrac{x}{1 + e^{x^2}} \, dx$, Formula 37

In Exercises 9–34, use the table of integrals in this section to find the indefinite integral.

9. $\int \dfrac{1}{x(1 + x)} \, dx$

10. $\int \dfrac{1}{x(1 + x)^2} \, dx$

11. $\int \dfrac{1}{x\sqrt{x^2 + 9}} \, dx$

12. $\int \dfrac{1}{\sqrt{x^2 - 1}} \, dx$

13. $\int \dfrac{1}{x\sqrt{4 - x^2}} \, dx$

14. $\int \dfrac{\sqrt{x^2 - 9}}{x^2} \, dx$

15. $\int x \ln x \, dx$

16. $\int x^2(\ln x^3)^2 \, dx$

17. $\int \dfrac{6x}{1 + e^{3x^2}} \, dx$

18. $\int \dfrac{1}{1 + e^x} \, dx$

19. $\int x\sqrt{x^4 - 4} \, dx$

20. $\int \dfrac{x}{x^4 - 9} \, dx$

21. $\int \dfrac{t^2}{(2 + 3t)^3} \, dt$

22. $\int \dfrac{\sqrt{3 + 4t}}{t} \, dt$

23. $\int \dfrac{s}{s^2\sqrt{3 + s}} \, ds$

24. $\int \sqrt{3 + x^2} \, dx$

25. $\int \dfrac{x^2}{(3 + 2x)^5} \, dx$

27. $\int \dfrac{1}{x^2\sqrt{1 - x^2}} \, d$

29. $\int x^2 \ln x \, dx$

31. $\int \dfrac{x^2}{(3x - 5)^2} \, dx$

33. $\int \dfrac{\ln x}{x(4 + 3 \ln x)}$

In Exercises 35–40, us of the region bounde a graphing utility to

35. $y = \dfrac{x}{\sqrt{x + 1}}$, y

36. $y = \dfrac{2}{1 + e^{4x}}$, y

37. $y = \dfrac{x}{1 + e^{x^2}}$, y

38. $y = \dfrac{-e^x}{1 - e^{2x}}$, y

39. $y = x^2\sqrt{x^2 + 4}$

40. $y = \dfrac{1}{\sqrt{x}(1 + 2}$

In Exercises 41–44, ev

41. $\int_0^5 \dfrac{x}{\sqrt{5 + 2x}} \, dx$

42. $\int_0^5 \dfrac{x}{(4 + x)^2} \, dx$

43. $\int_0^4 \dfrac{6}{1 + e^{0.5x}} \, dx$

44. $\int_1^4 x \ln x \, dx$

In Exercises 45–48, fir gration table and (b)

Integral

45. $\int x^2 e^x \, dx$

46. $\int x^4 \ln x \, dx$

47. $\int \dfrac{1}{x^2(x + 1)} \, dx$

48. $\int \dfrac{1}{x^2 - 75} \, dx$

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tegrals in this section to find

$\int \frac{1}{x(1+x)^2}\,dx$

$\int \frac{1}{\sqrt{x^2-1}}\,dx$

$\int \frac{\sqrt{x^2-9}}{x^2}\,dx$

$\int x^2(\ln x^3)^2\,dx$

$\int \frac{1}{1+e^x}\,dx$

$\int \frac{x}{x^4-9}\,dx$

$\int \frac{\sqrt{3+4t}}{t}\,dt$

$\int \sqrt{3+x^2}\,dx$

25. $\int \frac{x^2}{(3+2x)^5}\,dx$ **26.** $\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$

27. $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$ **28.** $\int \frac{2x}{(1-3x)^2}\,dx$

29. $\int x^2 \ln x\,dx$ **30.** $\int xe^{x^2}\,dx$

31. $\int \frac{x^2}{(3x-5)^2}\,dx$ **32.** $\int \frac{1}{2x^2(2x-1)^2}\,dx$

33. $\int \frac{\ln x}{x(4+3\ln x)}\,dx$ **34.** $\int (\ln x)^3\,dx$

In Exercises 35–40, use the integration table to find the exact area of the region bounded by the graphs of the equations. Then use a graphing utility to graph the region and approximate the area.

35. $y = \frac{x}{\sqrt{x+1}},\ y = 0,\ x = 8$

36. $y = \frac{2}{1+e^{4x}},\ y = 0,\ x = 0,\ x = 1$

37. $y = \frac{x}{1+e^{x^2}},\ y = 0,\ x = 2$

38. $y = \frac{-e^x}{1-e^{2x}},\ y = 0,\ x = 1,\ x = 2$

39. $y = x^2\sqrt{x^2+4},\ y = 0,\ x = \sqrt{5}$

40. $y = \frac{1}{\sqrt{x}(1+2\sqrt{x})},\ y = 0,\ x = 1,\ x = 4$

In Exercises 41–44, evaluate the definite integral.

41. $\int_0^5 \frac{x}{\sqrt{5+2x}}\,dx$

42. $\int_0^5 \frac{x}{(4+x)^2}\,dx$

43. $\int_0^4 \frac{6}{1+e^{0.5x}}\,dx$

44. $\int_1^4 x\ln x\,dx$

In Exercises 45–48, find the indefinite integral (a) using the integration table and (b) using the specified method.

	Integral	*Method*
45.	$\int x^2e^x\,dx$	Integration by parts
46.	$\int x^4 \ln x\,dx$	Integration by parts
47.	$\int \frac{1}{x^2(x+1)}\,dx$	Partial fractions
48.	$\int \frac{1}{x^2-75}\,dx$	Partial fractions

In Exercises 49–52, complete the square to express each polynomial as the sum or difference of squares.

49. (a) $x^2 + 6x$
(b) $x^2 - 8x + 9$
(c) $x^4 + 2x^2 - 5$
(d) $3 - 2x - x^2$

50. (a) $x^2 + 4x$
(b) $x^2 + 16x - 1$
(c) $x^4 + 8x^2 + 1$
(d) $9x^2 + 36x - 1$

51. (a) $4x^2 + 12x + 15$
(b) $3x^2 - 12x - 9$
(c) $x^2 - 2x$
(d) $9 + 8x - x^2$

52. (a) $16x^2 - 96x + 3$
(b) $x^2 + 4x - 1$
(c) $1 - 8x - x^2$
(d) $6x - x^2$

In Exercises 53–60, complete the square and then use the integration table to find the indefinite integral.

53. $\int \frac{1}{x^2+6x-8}\,dx$ **54.** $\int \frac{1}{x^2+4x-5}\,dx$

55. $\int \frac{1}{(x-1)\sqrt{x^2-2x+2}}\,dx$ **56.** $\int \sqrt{x^2-6x}\,dx$

57. $\int \frac{1}{2x^2-4x-6}\,dx$ **58.** $\int \frac{\sqrt{7-6x-x^2}}{x+3}\,dx$

59. $\int \frac{x}{\sqrt{x^4+2x^2+2}}\,dx$ **60.** $\int \frac{x\sqrt{x^4+4x^2+5}}{x^2+2}\,dx$

Population Growth In Exercises 61 and 62, use a graphing utility to graph the growth function. Use the table of integrals to find the average value of the growth function over the interval, where N is the size of a population and t is the time in days.

61. $N = \frac{50}{1+e^{4.8-1.9t}}$, $[3, 4]$

62. $N = \frac{375}{1+e^{4.20-0.25t}}$, $[21, 28]$

63. ***Revenue*** The revenue (in dollars per year) for a new product is modeled by

$$R = 10{,}000\left[1 - \frac{1}{(1+0.1t^2)^{1/2}}\right]$$

where t is the time in years. Estimate the total revenue from sales of the product over its first 2 years on the market.

64. ***Consumer and Producer Surpluses*** Find the consumer surplus and the producer surplus for a product with the given demand and supply functions.

Demand: $p = \frac{60}{\sqrt{x^2+81}}$, *Supply:* $p = \frac{x}{3}$

65. ***Profit*** The net profits P (in billions of dollars per year) for Hershey Foods from 2000 through 2003 can be modeled by

$P = \sqrt{0.04t - 0.3}$, $10 \le t \le 13$

where t is the time in years, with $t = 10$ corresponding to 2000. Find the average net profit over that time period. *(Source: Hershey Foods Corp.)*

PREREQUISITE REVIEW 6.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find the indicated derivative.

1. $f(x) = \frac{1}{x}$, $f''(x)$

2. $f(x) = \ln(2x + 1)$, $f^{(4)}(x)$

3. $f(x) = 2 \ln x$, $f^{(4)}(x)$

4. $f(x) = x^3 - 2x^2 + 7x - 12$, $f''(x)$

5. $f(x) = e^{2x}$, $f^{(4)}(x)$

6. $f(x) = e^{x^2}$, $f''(x)$

In Exercises 7 and 8, find the absolute maximum of f on the interval.

7. $f(x) = -x^2 + 6x + 9, [0, 4]$

8. $f(x) = \frac{8}{x^3}, [1, 2]$

In Exercises 9 and 10, solve for n.

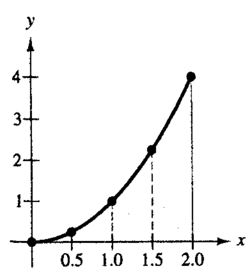
9. $\frac{1}{4n^2} < 0.001$

10. $\frac{1}{16n^4} < 0.0001$

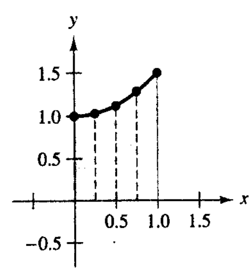
EXERCISES 6.5

In Exercises 1–12, use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the indicated value of n. Compare these results with the exact value of the definite integral. Round your answers to four decimal places.

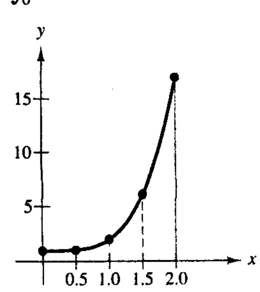
1. $\int_0^2 x^2\, dx,\ n = 4$



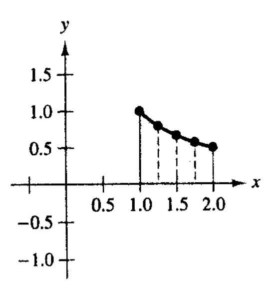
2. $\int_0^1 \left(\frac{x^2}{2} + 1\right) dx, n = 4$



3. $\int_0^2 (x^4 + 1)\, dx,\ n = 4$



4. $\int_1^2 \frac{1}{x}\, dx,\ n = 4$



5. $\int_0^2 x^3\, dx,\ n = 8$

6. $\int_1^3 (4 - x^2)\, dx,\ n = 4$

7. $\int_1^2 \frac{1}{x}\, dx,\ n = 8$

8. $\int_1^2 \frac{1}{x^2}\, dx,\ n = 4$

9. $\int_0^4 \sqrt{x}\, dx,\ n = 8$

10. $\int_0^2 \sqrt{1 + x}\, dx,\ n = 4$

11. $\int_0^1 \frac{1}{1 + x}\, dx,\ n = 4$

12. $\int_0^2 x\sqrt{x^2 + 1}\, dx,\ n = 4$

In Exercises 13–20, approximate the integral using (a) the Trapezoidal Rule and (b) Simpson's Rule. (Round your answers to three significant digits.)

	Definite Integral	*Subdivisions*
13.	$\int_0^1 \frac{1}{1+x^2}\,dx$	$n = 4$
14.	$\int_0^2 \frac{1}{\sqrt{1+x^3}}\,dx$	$n = 4$
15.	$\int_0^1 \sqrt{1-x^2}\,dx$	$n = 4$
16.	$\int_0^1 \sqrt{1-x^2}\,dx$	$n = 8$
17.	$\int_0^2 e^{-x^2}\,dx$	$n = 2$
18.	$\int_0^2 e^{-x^2}\,dx$	$n = 4$
19.	$\int_0^3 \frac{1}{2-2x+x^2}\,dx$	$n = 6$
20.	$\int_0^3 \frac{x}{2+x+x^2}\,dx$	$n = 6$

Present Value In Exercises 21 and 22, use a program similar to the Simpson's Rule program on page 430 with $n = 8$ to approximate the present value of the income $c(t)$ over t_1 years at the given annual interest rate r. Then use the integration capabilities of a graphing utility to approximate the present value. Compare the results. (Present value is defined in Section 6.2.)

21. $c(t) = 6000 + 200\sqrt{t},\ r = 7\%,\ t_1 = 4$

22. $c(t) = 200{,}000 + 15{,}000\sqrt[3]{t},\ r = 10\%,\ t_1 = 8$

Marginal Analysis In Exercises 23 and 24, use a program similar to the Simpson's Rule program on page 430 with $n = 4$ to approximate the change in revenue from the marginal revenue function dR/dx. In each case, assume that the number of units sold x increases from 14 to 16.

23. $\frac{dR}{dx} = 5\sqrt{8000 - x^3}$

24. $\frac{dR}{dx} = 50\sqrt{x}\sqrt{20 - x}$

Probability In Exercises 25–28, use a program similar to the Simpson's Rule program on page 430 with $n = 6$ to approximate the indicated normal probability. The standard normal probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}.$$

If x is chosen at random from a population with this density, then the probability that x lies in the interval $[a, b]$ is

$$P(a \le x \le b) = \int_a^b f(x)\,dx.$$

25. $P(0 \le x \le 1)$

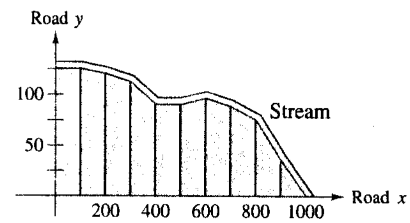
26. $P(0 \le x \le 2)$

27. $P(0 \le x \le 4)$

28. $P(0 \le x \le 1.5)$

Surveying In Exercises 29 and 30, use a program similar to the Simpson's Rule program on page 430 to estimate the number of square feet of land in the lot, where x and y are measured in feet, as shown in the figures. In each case, the land is bounded by a stream and two straight roads.

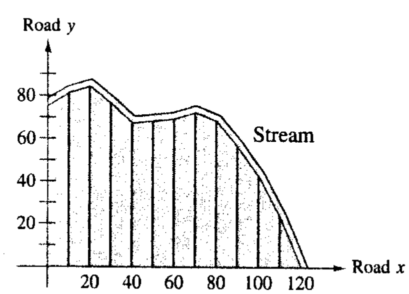
29.



x	0	100	200	300	400	500
y	125	125	120	112	90	90

x	600	700	800	900	1000
y	95	88	75	35	0

30.



x	0	10	20	30	40	50	60
y	75	81	84	76	67	68	69

x	70	80	90	100	110	120
y	72	68	56	42	23	0

In Exercises 31–
error in approxim
and (b) Simpson

31. $\int_0^2 x^4\,dx$

33. $\int_0^1 e^{x^3}\,dx$

In Exercises 35–
error in the app
0.0001 using (a)

35. $\int_0^1 x^4\,dx$

37. $\int_1^3 e^{2x}\,dx$

In Exercises 39–4
page 430 to appr

39. $\int_1^4 x\sqrt{x+}$

41. $\int_2^5 10xe^{-x}\,d$

43. Prove that S
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demonstrate

$\int_0^1 x^3\,dx,$

44. Use a progr
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$y = x\sqrt[3]{x}$

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In Exercises 45 an
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$\int_b^a \sqrt{1 + [f}$

45. ***Arc Length***
feet long is in

$y = \frac{x^2}{800}($

Use a progra
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using the tab
integration.

In Exercises 31–34, use the error formulas to find bounds for the error in approximating the integral using (a) the Trapezoidal Rule and (b) Simpson's Rule. (Let $n = 4$.)

31. $\int_0^2 x^4\,dx$ **32.** $\int_0^1 \frac{1}{x+1}\,dx$

33. $\int_0^1 e^{x^3}\,dx$ **34.** $\int_0^1 e^{-x^2}\,dx$

In Exercises 35–38, use the error formulas to find n such that the error in the approximation of the definite integral is less than 0.0001 using (a) the Trapezoidal Rule and (b) Simpson's Rule.

35. $\int_0^1 x^4\,dx$ **36.** $\int_1^3 \frac{1}{x}\,dx$

37. $\int_1^3 e^{2x}\,dx$ **38.** $\int_3^5 \ln x\,dx$

In Exercises 39–42, use the program for Simpson's Rule given on page 430 to approximate the integral. Use $n = 100$.

39. $\int_1^4 x\sqrt{x+4}\,dx$ **40.** $\int_1^4 x^2\sqrt{x+4}\,dx$

41. $\int_2^5 10xe^{-x}\,dx$ **42.** $\int_2^5 10x^2e^{-x}\,dx$

43. Prove that Simpson's Rule is exact when used to approximate the integral of a cubic polynomial function, and demonstrate the result for

$$\int_0^1 x^3\,dx, \qquad n = 2.$$

44. Use a program similar to the Simpson's Rule program on page 430 with $n = 4$ to find the volume of the solid generated by revolving the region bounded by the graphs of

$$y = x\sqrt[3]{x+4}, \quad y = 0, \quad \text{and} \quad x = 4$$

about the x-axis.

In Exercises 45 and 46, use the definite integral below to find the required arc length. If f has a continuous derivative, then the arc length of f between the points $(a, f(a))$ and $(b, f(b))$ is

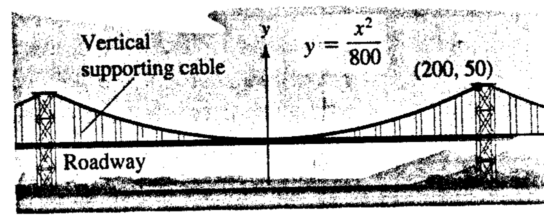
$$\int_b^a \sqrt{1 + [f'(x)]^2}\,dx.$$

45. *Arc Length* The suspension cable on a bridge that is 400 feet long is in the shape of a parabola whose equation is

$$y = \frac{x^2}{800} \text{ (see figure).}$$

Use a program similar to the Simpson's Rule program on page 430 with $n = 12$ to approximate the length of the cable. Compare this result with the length obtained by using the table of integrals in Section 6.4 to perform the integration.

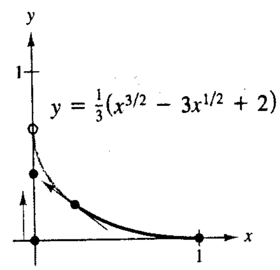
Figure for 45



46. *Arc Length* A fleeing hare leaves its burrow $(0, 0)$ and moves due north (up the y-axis). At the same time, a pursuing lynx leaves from 1 yard east of the burrow $(1, 0)$ and always moves toward the fleeing hare (see figure). If the lynx's speed is twice that of the hare's, the equation of the lynx's path is

$$y = \frac{1}{3}(x^{3/2} - 3x^{1/2} + 2).$$

Find the distance traveled by the lynx by integrating over the interval $[0, 1]$.



47. *Medicine* A body assimilates a 12-hour cold tablet at a rate modeled by

$$\frac{dC}{dt} = 8 - \ln(t^2 - 2t + 4), \qquad 0 \le t \le 12$$

where dC/dt is measured in milligrams per hour and t is the time in hours. Find the total amount of the drug absorbed into the body during the 12 hours.

48. *Medicine* The concentration M (in grams per liter) of a 6-hour allergy medicine in a body is modeled by

$$M = 12 - 4\ln(t^2 - 4t + 6), \qquad 0 \le t \le 6$$

where t is the time in hours since the allergy medication was taken. Find the average level of concentration in the body over the six-hour period.

49. *Consumer Trends* The rate of change S in the number of subscribers to a newly introduced magazine is modeled by

$$\frac{dS}{dt} = 1000t^2e^{-t}, \qquad 0 \le t \le 6$$

where t is the time in years. Find the total increase in the number of subscribers during the first 6 years.

PREREQUISITE REVIEW 6.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find the limit.

1. $\lim_{x\to 2} (2x + 5)$

2. $\lim_{x\to 1} \left(\frac{1}{x} + 2x^2\right)$

3. $\lim_{x\to -4} \frac{x + 4}{x^2 - 16}$

4. $\lim_{x\to 0} \frac{x^2 - 2x}{x^3 + 3x^2}$

5. $\lim_{x\to 1} \frac{1}{\sqrt{x - 1}}$

6. $\lim_{x\to -3} \frac{x^2 + 2x - 3}{x + 3}$

In Exercises 7–10, evaluate the expression (a) when $x = b$ and (b) when $x = 0$.

7. $\frac{4}{3}(2x - 1)^3$

8. $\frac{1}{x - 5} + \frac{3}{(x - 2)^2}$

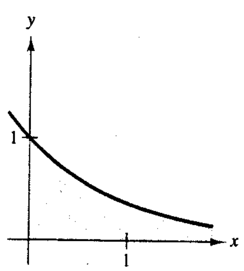
9. $\ln(5 - 3x^2) - \ln(x + 1)$

10. $e^{3x^2} + e^{-3x^2}$

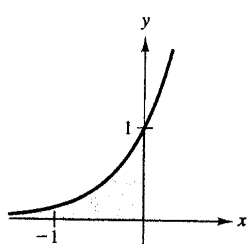
EXERCISES 6.6

In Exercises 1–14, determine whether or not the improper integral converges. If it does, evaluate the integral.

1. $\int_0^{\infty} e^{-x}\,dx$



2. $\int_{-\infty}^{0} e^{2x}\,dx$



3. $\int_1^{\infty} \frac{1}{x^2}\,dx$

4. $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$

5. $\int_0^{\infty} e^{x/3}\,dx$

6. $\int_0^{\infty} \frac{5}{e^{2x}}\,dx$

7. $\int_5^{\infty} \frac{x}{\sqrt{x^2 - 16}}\,dx$

8. $\int_{1/2}^{\infty} \frac{1}{\sqrt{2x - 1}}\,dx$

9. $\int_{-\infty}^{0} e^{-x}\,dx$

10. $\int_{-\infty}^{-1} \frac{1}{x^2}\,dx$

11. $\int_1^{\infty} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

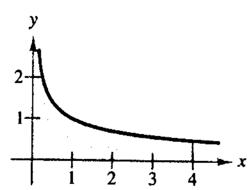
12. $\int_{-\infty}^{0} \frac{x}{x^2 + 1}\,dx$

13. $\int_{-\infty}^{\infty} 2xe^{-3x^2}\,dx$

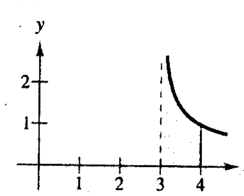
14. $\int_{-\infty}^{\infty} x^2e^{-x^3}\,dx$

In Exercises 15–18, determine the divergence or convergence of the improper integral. Evaluate the integral if it converges.

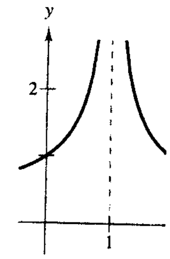
15. $\int_0^{4} \frac{1}{\sqrt{x}}\,dx$



16. $\int_3^{4} \frac{1}{\sqrt{x - 3}}\,dx$



17. $\int_0^{2} \frac{1}{(x - 1)^{2/3}}$ a



In Exercises 19–28, e

19. $\int_0^{1} \frac{1}{1 - x}\,dx$

21. $\int_0^{9} \frac{1}{\sqrt{9 - x}}\,dx$

23. $\int_0^{1} \frac{1}{x^2}\,dx$

25. $\int_0^{2} \frac{1}{\sqrt[3]{x - 1}}\,dx$

26. $\int_0^{2} \frac{1}{(x - 1)^{4/3}}$ d

27. $\int_3^{4} \frac{1}{\sqrt{x^2 - 9}}\,dx$

28. $\int_3^{5} \frac{1}{x^2\sqrt{x^2 - 9}}$

In Exercises 29 and 3(the graphs of the giv solid generated by re

29. $y = \frac{1}{x^2}, y = 0, x$

30. $y = e^{-x}, y = 0,$

In Exercises 31–34, co a and n to demonstra

$$\lim_{x\to\infty} x^n e^{-ax} = 0$$

x	1	10
$x^n e^{-ax}$		

31. $a = 1, n = 1$

32. $a = 2, n = 4$

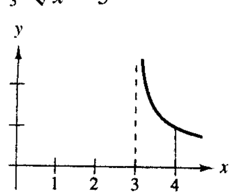
33. $a = \frac{1}{2}, n = 2$

34. $a = \frac{1}{2}, n = 5$

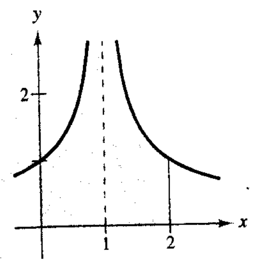
er sections. You will

gence or convergence of
gral if it converges.

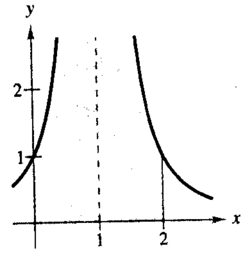
$\int_3^4 \frac{1}{\sqrt{x-3}}\,dx$



17. $\int_0^2 \frac{1}{(x-1)^{2/3}}\,dx$



18. $\int_0^2 \frac{1}{(x-1)^2}\,dx$



In Exercises 19–28, evaluate the improper integral.

19. $\int_0^1 \frac{1}{1-x}\,dx$

20. $\int_0^{27} \frac{5}{\sqrt[3]{x}}\,dx$

21. $\int_0^9 \frac{1}{\sqrt{9-x}}\,dx$

22. $\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx$

23. $\int_0^1 \frac{1}{x^2}\,dx$

24. $\int_0^1 \frac{1}{x}\,dx$

25. $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$

26. $\int_0^2 \frac{1}{(x-1)^{4/3}}\,dx$

27. $\int_3^4 \frac{1}{\sqrt{x^2-9}}\,dx$

28. $\int_3^5 \frac{1}{x^2\sqrt{x^2-9}}\,dx$

In Exercises 29 and 30, (a) find the area of the region bounded by the graphs of the given equations and (b) find the volume of the solid generated by revolving the region about the x-axis.

29. $y = \frac{1}{x^2}, y = 0, x \geq 1$

30. $y = e^{-x}, y = 0, x \geq 0$

In Exercises 31–34, complete the table for the specified values of a and n to demonstrate that

$$\lim_{x\to\infty} x^n e^{-ax} = 0, \qquad a > 0, n > 0.$$

x	1	10	25	50
$x^n e^{-ax}$				

31. $a = 1, n = 1$

32. $a = 2, n = 4$

33. $a = \frac{1}{2}, n = 2$

34. $a = \frac{1}{2}, n = 5$

In Exercises 35–38, use the results of Exercises 31–34 to evaluate the improper integral.

35. $\int_0^\infty x^2 e^{-x}\,dx$

36. $\int_0^\infty (x-1)e^{-x}\,dx$

37. $\int_0^\infty xe^{-2x}\,dx$

38. $\int_0^\infty xe^{-x}\,dx$

39. ***Present Value*** A business is expected to yield a continuous flow of profit at the rate of \$500,000 per year. If money will earn interest at the nominal rate of 9% per year compounded continuously, what is the present value of the business (a) for 20 years and (b) forever? (Present value is defined in Section 6.2.)

40. ***Present Value*** Repeat Exercise 39 for a farm that is expected to produce a profit of \$75,000 per year. Assume that money will earn interest at the nominal rate of 8% compounded continuously. (Present value is defined in Section 6.2.)

Capitalized Cost In Exercises 41 and 42, find the capitalized cost C of an asset (a) for $n = 5$ years, (b) for $n = 10$ years, and (c) forever. The capitalized cost is given by

$$C = C_0 + \int_0^n c(t)e^{-rt}dt$$

where C_0 is the original investment, t is the time in years, r is the annual interest rate compounded continuously, and $c(t)$ is the annual cost of maintenance. [*Hint:* For part (c), see Exercises 31–34.]

41. $C_0 = \$650{,}000, c(t) = 25{,}000, r = 10\%$

42. $C_0 = \$650{,}000, c(t) = \$25{,}000(1 + 0.08t), r = 12\%$

43. ***Women's Height*** The mean height of American women between the ages of 25 and 34 is 64.5 inches, and the standard deviation is 2.4 inches. Find the probability that a 25- to 34-year-old woman chosen at random is
    (a) between 5 and 6 feet tall.
    (b) 5 feet 8 inches or taller.
    (c) 6 feet or taller.
    (*Source: U.S. National Center for Health Statistics*)

44. ***Quality Control*** A company manufactures wooden yardsticks. The lengths of the yardsticks are normally distributed with a mean of 36 inches and a standard deviation of 0.2 inch. Find the probability that a yardstick is
    (a) longer than 35.5 inches.
    (b) longer than 35.9 inches.