Math 16B (Winter 2008)
Kouba
Exam 1

Please PRINT your name here : KEY

Your Exam ID Number ____________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO TAKE AN EXAM FOR ANOTHER PERSON. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive little or no credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 6 pages, including the cover page.

6. Include units on answers where units are appropriate.

7. You have until 8:50 a.m. sharp to finish the exam.

1.) (2 pts. each) Determine whether each statement is true (T) or false (F). Then circle the appropriate response. Assume that x and y are psoitive numbers.

a.) $\ln x + \ln y = \ln(x+y)$ T F

b.) $e^x + e^y = (e^x)^y$ T F

c.) $\ln x \cdot \ln y = \ln(xy)$ T F

d.) $e^{x-y} = \dfrac{e^x}{e^y}$ T F

2.) (8 pts.) Cesium-137 is an isotope produced by nuclear fission, and is used in medical radiation therapy devices for treating cancer. It's half-life is about 30.17 years. If a sample of Cs-137 has an initial mass of 100 mg., how much will remain after 100 years ?

assume $A = Ce^{kt}$, $C = 100$ mg. $\rightarrow A = 100e^{kt}$

and $t = 30.17$ yrs. $\rightarrow A = 50$ mg. $\rightarrow 50 = 100e^{30.17k}$

$\rightarrow \frac{1}{2} = e^{30.17k} \rightarrow \ln(1/2) = 30.17k \rightarrow k = \frac{\ln(1/2)}{30.17} \rightarrow$

$\underline{A = 100e^{\frac{\ln(1/2)}{30.17}t}}$; let $t = 100$ yrs. $\rightarrow A = 100e^{\frac{\ln(1/2)}{30.17}(100)} \approx 10.05$ mg.

3.) A watermelon is dropped from a helicopter hovering at 1600 ft. above the ground. Assume that the acceleration due to gravity is -32 ft./sec.^{2} .

a.) (4 pts.) Derive formulas for the velocity and height (above the ground) of the doomed watermelon.

$s''(t) = -32 \rightarrow s'(t) = -32t + C$ $(t=0,\ s'=0) \rightarrow$

$0 = 0 + C \rightarrow C = 0 \rightarrow$ vel.: $\boxed{s'(t) = -32t} \rightarrow$

$s(t) = -16t^2 + C$ $(t=0,\ s=1600) \rightarrow 1600 = 0 + C \rightarrow C = 1600$

$\rightarrow$ hgt.: $\boxed{s(t) = 1600 - 16t^2}$

b.) (2 pts.) In how many seconds will the watermelon strike the ground ?

strike ground: $s(t) = 0 \rightarrow 1600 - 16t^2 = 0 \rightarrow$

$16t^2 = 1600 \rightarrow t^2 = 100 \rightarrow \boxed{t = 10 \text{ sec.}}$

c.) (2 pts.) What is the watermelon's velocity as it strikes the ground ?

$s'(10) = -32(10) = -320$ ft./sec.

4.) (7 pts.) Find a function $f(x)$ which has the following properties : $f''(x) = 20x^3 + 2$, $f'(0) = -1$, and $f(1) = 3$

$\longrightarrow f'(x) = 5x^4 + 2x + C \quad (x=0,\ f'(0) = -1) \longrightarrow$

$-1 = 0 + 0 + C \rightarrow C = -1 \rightarrow f'(x) = 5x^4 + 2x - 1$

$\longrightarrow f(x) = x^5 + x^2 - x + C \quad (x=1,\ f(1) = 3) \longrightarrow$

$3 = 1 + \cancel{1} - \cancel{1} + C \longrightarrow C = 2 \longrightarrow$

$\boxed{f(x) = x^5 + x^2 - x + 2}$

5.) (6 pts.) Let $y = x^3 \ln x$. Solve $y' = 0$ for x .

$\xrightarrow{D} y' = x^3 \cdot \frac{1}{x} + 3x^2 \ln x = x^2 + 3x^2 \ln x$

$= x^2(1 + 3\ln x) \rightarrow \cancel{x^2=0}$ (NO) or

$1 + 3\ln x = 0 \longrightarrow 3\ln x = -1 \rightarrow \ln x = -1/3 \longrightarrow$

$\boxed{x = e^{-1/3}}$

6.) (9 pts.) Let $f(x) = xe^x$. Set up a sign chart for the second derivative, $f''(x)$, to determine any inflection points for the graph of f .

$\xrightarrow{D} f'(x) = xe^x + 1 \cdot e^x \xrightarrow{D} f''(x) = xe^x + 1 \cdot e^x + e^x$

$= xe^x + 2e^x = e^x(x+2) = 0 \longrightarrow \cancel{e^x=0}$ (NO) or

$x = -2$

$$\begin{array}{ccc} - & 0 & + \\ \hline & x=-2 & \end{array} \quad f''$$

$\left.\begin{array}{l} x = -2 \\ y = -2e^{-2} \end{array}\right\}$ infl. pt.

7.) (7 pts.) You deposit \$5000 in a savings account earning an annual interest rate of 5.5% compounded monthly. How long will it take for your account to double in size ?

assume $A = P\left(1+\frac{r}{n}\right)^{nt}$, $P = \$5000$, $r = 0.055$, $n = 12$, $A = \$10{,}000$ $\longrightarrow$

$$10{,}000 = 5000\left(1+\frac{0.055}{12}\right)^{12t} \rightarrow 2 = \left(1+\frac{0.055}{12}\right)^{12t}$$

$$\rightarrow \ln 2 = \ln\left(1+\frac{0.055}{12}\right)^{12t} = 12t \cdot \ln\left(1+\frac{0.055}{12}\right)$$

$$\rightarrow t = \frac{\ln 2}{12 \ln\left(1+\frac{0.055}{12}\right)} \approx 12.63 \text{ yrs.}$$

8.) (8 pts.) Use implicit differentiation to determine $y' = \frac{dy}{dx}$ for $3x + y^2 \cdot 5^x = \ln y$.

$$3X + Y^2 \cdot 5^X = \ln Y \xrightarrow{D} 3 + Y^2 \cdot 5^X \ln 5 + 2YY' \cdot 5^X = \frac{1}{Y} \cdot Y'$$

$$\rightarrow 2YY' \cdot 5^X - \frac{1}{Y} Y' = -3 - Y^2 \cdot 5^X \ln 5$$

$$\rightarrow Y'\left(2Y \cdot 5^X - \frac{1}{Y}\right) = -3 - Y^2 \cdot 5^X \ln 5$$

$$\rightarrow Y' = \frac{-3 - Y^2 \cdot 5^X \cdot \ln 5}{2Y \cdot 5^X - 1/Y}$$

9.) (7 pts.) Determine if the following antiderivative is true (T) or false (F). Give supporting steps or reasons for your answer.

$$\int \frac{1-x^2}{(1+x^2)^2}\,dx = \frac{x}{1+x^2} + C$$

T since

$$D\left(\frac{X}{1+X^2}\right) = \frac{(1+X^2)(1) - X \cdot 2X}{(1+X^2)^2} = \frac{1-X^2}{(1+X^2)^2}$$

10.) (8 pts. each) Determine the following antiderivatives.

a.) $\int x(3+\sqrt{x})\ dx \quad = \int \left(3x + x^{3/2}\right) dx$

$= \frac{3}{2}x^2 + \frac{2}{5}x^{5/2} + c$

b.) $\int \frac{2x}{(1+x^2)^3}\ dx \quad$ (Let $u = 1+x^2 \rightarrow du = 2x\ dx$)

$= \int \frac{1}{u^3}\ du = \int u^{-3} du = \frac{u^{-2}}{-2} + c = \frac{(1+x^2)^{-2}}{-2} + c$

c.) $\int \sqrt{5x+34}\ dx \quad$ (Let $u = 5x+34 \rightarrow du = 5\,dx$
$\rightarrow \frac{1}{5} du = dx$)

$= \frac{1}{5}\int \sqrt{u}\ du = \frac{1}{5}\cdot\frac{2}{3}u^{3/2} + c$

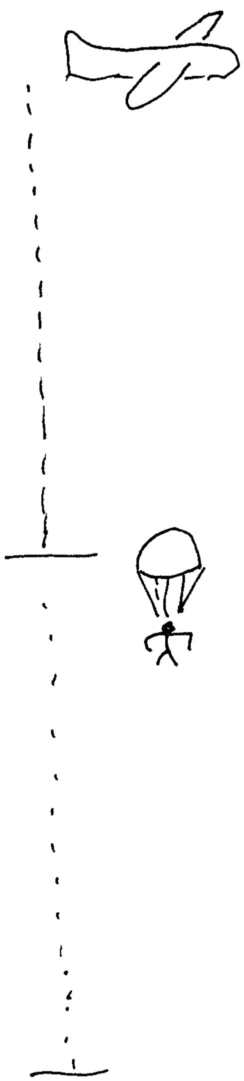
$= \frac{2}{15}(5x+34)^{3/2} + c$

d.) $\int (x\sec^2 x + \tan x)\cdot(x\tan x)^2\ dx \quad$ (Let $u = x\tan x \rightarrow$
$du = (x\sec^2 x + \tan x)\ dx$)

$= \int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}(x\tan x)^3 + c$

EXTRA CREDIT PROBLEM The following problem is worth 10 points. This problem is OPTIONAL.

1.) A parachutist jumps from a plane at an altitude of 2000 feet and free falls for 10 seconds. At that point the parachute opens and the the parachutist floats to earth at a constant rate of 10 ft./sec. From the moment the parachutist jumps from the plane, how much time will it take to reach the ground ?



$s''(t) = -32 \rightarrow s'(t) = -32t \rightarrow$

$s(t) = -16t^2 + 2000$ (height above ground)

$s(10) = -16(100) + 2000 = 400$ ft ;

$\frac{400 \text{ ft.}}{10 \text{ ft./sec.}} = 40$ sec. , then

total time is

$10 + 40 = 50$ sec.