

Math 16B

Exam 1 Solutions

- 1.) a.) F b.) T c.) T d.) F e.) F
 f.) F g.) F h.) F i.) T j.) T

2.) Assume $A = Ce^{kt}$; $t = 5730$ yrs. and
 $A = \frac{1}{2}C \rightarrow \frac{1}{2}C = C e^{5730k} \rightarrow \ln \frac{1}{2} = 5730k \rightarrow$
 $k = \frac{\ln \frac{1}{2}}{5730}$ so that $A = C e^{\frac{\ln \frac{1}{2}}{5730} t}$; if
 $A = 0.085C \rightarrow 0.085C = C e^{\frac{\ln \frac{1}{2}}{5730} t} \rightarrow$
 $\ln 0.085 = \frac{\ln \frac{1}{2}}{5730} t \rightarrow t = \frac{5730 \ln 0.085}{\ln \frac{1}{2}} \approx 20,378$ yrs.

3.) $y = \frac{8e^{-x}}{x+4} \Rightarrow y' = \frac{(x+4) \cdot 8e^{-x}(-1) - 8e^{-x}(1)}{(x+4)^2}$

and $x=0 \rightarrow y = \frac{8 \cdot e^0}{0+4} = \frac{8(1)}{4} = 2$, and

$y' = \frac{(4)(8)(1)(-1) - 8(1)}{(4)^2} = \frac{-40}{16} = -\frac{5}{2}$ so +

slope is $m = \frac{2}{5}$, then \perp line is

$y - 2 = \frac{2}{5}(x - 0)$ or $y = \frac{2}{5}x + 2$

4.) $f''(x) = 2x + 12x^2 \rightarrow f' = x^2 + 4x^3 + C$ and

$f'(0) = 10$ so $C = 10$ and $f'(x) = x^2 + 4x^3 + 10 \rightarrow$

$f(x) = \frac{1}{3}x^3 + x^4 + 10x + C$ and $f(0) = 50$ so $C = 50 \rightarrow$

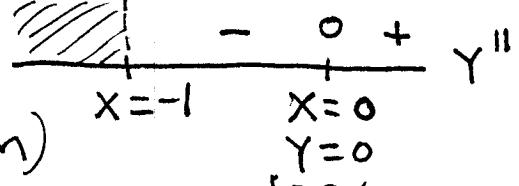
$f(x) = \frac{1}{3}x^3 + x^4 + 10x + 50$

5.) $f(x) = x^2 e^{-x} \rightarrow f'(x) = -x^2 e^{-x} + 2x e^{-x} = x e^{-x}(2-x) = 0$

$\begin{array}{r} - \\ \text{rel. } \left\{ \begin{array}{l} x=0 \\ Y=0 \end{array} \right. \end{array} \quad \begin{array}{r} + \\ \text{rel. } \left\{ \begin{array}{l} x=2 \\ Y=4e^{-2} \end{array} \right. \end{array} \quad \begin{array}{r} - \\ \text{rel. max.} \end{array}$

$$6.) Y = x^2 + 2 \ln(x+1) \rightarrow Y' = 2x + \frac{2}{x+1} = 2x + 2(x+1)^{-1} \rightarrow$$

$$Y'' = 2 - 2(x+1)^{-2} = 2 - \frac{2}{(x+1)^2} = 0$$


 A horizontal number line with points labeled \$x=-1\$, \$x=0\$, and \$Y=0\$. To the left of \$x=-1\$, there is a hatched rectangle above the line, indicating a negative value. Between \$x=-1\$ and \$x=0\$, the line is solid below the axis, indicating a positive value. To the right of \$x=0\$, the line is solid above the axis, indicating a positive value. There is a bracket under the line between \$x=0\$ and \$Y=0\$ labeled '\$0+\$'. Above the line, there is a bracket labeled '\$-\$'.

$$\rightarrow x=0 \text{ or } x \neq -2 \text{ (not in domain)}$$

$$7.) 2^x + \log_3 Y = Y^3 + x^2 \rightarrow$$

$$2^x \ln 2 + \frac{1}{\ln 3} \cdot \frac{1}{Y} \cdot Y' = 3Y^2 \cdot Y' + 2x \rightarrow$$

$$\frac{1}{\ln 3} \cdot \frac{1}{Y} \cdot Y' - 3Y^2 \cdot Y' = 2x - 2^x \ln 2 \rightarrow$$

$$Y' = \frac{2x - 2^x \ln 2}{\frac{1}{\ln 3} \cdot \frac{1}{Y} - 3Y^2}$$

$$8.) \int f(x) dx = \sin(7x) + C \rightarrow D(\sin(7x) + C) = f(x)$$

$$\rightarrow 7 \cos(7x) = f(x) \rightarrow f'(x) = -7^2 \sin(7x) \rightarrow$$

$$f''(x) = -7^3 \cos(7x)$$

$$9.) A = P(1 + \frac{r}{n})^{nt}, A = Pe^{rt} \rightarrow$$

acet. AA $\hookrightarrow A = 500 \left(1 + \frac{0.07}{12}\right)^{12t}$, acet. BB $A = 400 e^{0.08t}$

- a.) $t = 10$ yrs. \rightarrow (AA) has \$1004.83 and BB has \$890.22, difference: \$114.61
- b.) $t = 50$ yrs. \rightarrow AA has \$16,390.21 and (BB) has \$21,839.26, difference: \$5449.05

$$10.) a.) \int x^5(x^2 - 1) dx = \int (x^7 - x^5) dx$$

$$= \frac{1}{8} x^8 - \frac{1}{6} x^6 + C$$

$$b.) \int x(x^2-1)^5 dx \quad (\text{Let } u = x^2-1 \rightarrow du = 2x dx)$$

$$= \frac{1}{2} \int u^5 du \quad \rightarrow \frac{1}{2} du = x dx$$

$$= \frac{1}{2} \cdot \frac{1}{6} u^6 + C = \frac{1}{12} (x^2-1)^6 + C$$

$$c.) \int (7x+1)^{3/2} dx \quad (\text{Let } u = 7x+1 \rightarrow du = 7 dx)$$

$$= \frac{1}{7} \int u^{3/2} du \quad \rightarrow \frac{1}{7} du = dx$$

$$= \frac{1}{7} \cdot \frac{1}{\frac{5}{2}} u^{5/2} + C = \frac{2}{35} (7x+1)^{5/2} + C$$

$$d.) \int \frac{e^x}{\sqrt{4+e^x}} dx \quad (\text{Let } u = 4+e^x \rightarrow du = e^x dx)$$

$$= \int \frac{1}{\sqrt{u}} du = \int u^{-1/2} du = \frac{1}{\frac{1}{2}} u^{1/2} + C = 2(4+e^x)^{1/2} + C$$

Extra Credit

$$1.) Y = 2 \ln(3x+1) - \ln(2-x)^2 \rightarrow (\text{switch})$$

$$X = 2 \ln(3Y+1) - \ln(2-Y)^2 \rightarrow$$

$$X = \ln(3Y+1)^2 - \ln(2-Y)^2 \rightarrow X = \ln \frac{(3Y+1)^2}{(2-Y)^2} \rightarrow$$

$$X = \ln \left(\frac{3Y+1}{2-Y} \right)^2 \rightarrow \left(\frac{3Y+1}{2-Y} \right)^2 = e^X \rightarrow \frac{3Y+1}{2-Y} = e^{X/2} \rightarrow$$

$$3Y+1 = 2e^{X/2} - Ye^{X/2} \rightarrow 3Y + Ye^{X/2} = -1 + 2e^{X/2} \rightarrow$$

$$Y(3+e^{X/2}) = -1 + 2e^{X/2} \rightarrow Y = \frac{-1 + 2e^{X/2}}{3 + e^{X/2}} \quad (\text{inverse})$$

$$2.) \text{assume } A = Ce^{kt} \rightarrow A = 2358e^{kt} \text{ and}$$

$$t=10, A=4000 \rightarrow 4000 = 2358 e^{10k} \rightarrow$$

$$k = \frac{1}{10} \ln \frac{4000}{2358} \rightarrow \boxed{A = 2358 e^{\frac{1}{10} \ln \frac{4000}{2358} \cdot t}}, ;$$

100 yrs. ago : let $t = -100$! $\rightarrow A \approx 12$ people!