Math 16B
Exam 1 Solutions

1.) a.) F b.) T c.) T d.) F e.) F
f.) F g.) F h.) F i.) T j.) T

2.) assume $A = Ce^{kt}$; $t = 5730$ yrs. and
$A = \frac{1}{2}C \rightarrow \frac{1}{2}\cancel{C} = \cancel{C}e^{5730k} \rightarrow \ln\frac{1}{2} = 5730k \rightarrow$
$k = \frac{\ln 1/2}{5730}$ so that $A = Ce^{\frac{\ln 1/2}{5730}t}$; if
$A = 0.085C \rightarrow 0.085\cancel{C} = \cancel{C}e^{\frac{\ln 1/2}{5730}t} \rightarrow$
$\ln 0.085 = \frac{\ln 1/2}{5730}t \rightarrow t = \frac{5730 \ln 0.085}{\ln 1/2} \approx 20{,}378$ yrs.

3.) $Y = \frac{8e^{-x}}{x+4} \xrightarrow{D} Y' = \frac{(x+4)\cdot 8e^{-x}(-1) - 8e^{-x}(1)}{(x+4)^2}$

and $x = 0 \rightarrow Y = \frac{8\cdot e^0}{0+4} = \frac{8(1)}{4} = 2$, and
$Y' = \frac{(4)(8)(1)(-1) - 8(1)}{(4)^2} = \frac{-40}{16} = \frac{-5}{2}$ so $\perp$
slope is $m = \frac{2}{5}$, then $\perp$ line is
$Y - 2 = \frac{2}{5}(x-0)$ or $Y = \frac{2}{5}x + 2$

4.) $f''(x) = 2x + 12x^2 \rightarrow f' = x^2 + 4x^3 + C$ and
$f'(0) = 10$ so $C = 10$ and $f'(x) = x^2 + 4x^3 + 10 \rightarrow$
$f(x) = \frac{1}{3}x^3 + x^4 + 10x + C$ and $f(0) = 50$ so $C = 50 \rightarrow$
$f(x) = \frac{1}{3}x^3 + x^4 + 10x + 50$

5.) $f(x) = x^2e^{-x} \rightarrow f'(x) = -x^2e^{-x} + 2xe^{-x} = xe^{-x}(2-x) = 0$

$-\quad 0 \quad +\quad 0 \quad -$ f'

rel. min. $\left\{ \begin{array}{l} x=0 \\ Y=0 \end{array} \right.$ $\left. \begin{array}{l} x=2 \\ Y=4e^{-2} \end{array} \right\}$ rel. max.

6.) $Y = x^2 + 2\ln(x+1) \to Y' = 2x + \frac{2}{x+1} = 2x + 2(x+1)^{-1} \to$

$Y'' = 2 - 2(x+1)^{-2} = 2 - \frac{2}{(x+1)^2} = 0$

$\to x = 0$ or $x \neq -2$ (not in domain)

$-$ 0 $+$ Y''

$x=-1$ $x=0$

$Y=0$

infl. pt.

7.) $2^x + \log_3 Y = Y^3 + x^2 \xrightarrow{D}$

$2^x \ln 2 + \frac{1}{\ln 3} \cdot \frac{1}{Y} \cdot Y' = 3Y^2 \cdot Y' + 2x \to$

$\frac{1}{\ln 3} \cdot \frac{1}{Y} \cdot Y' - 3Y^2 \cdot Y' = 2x - 2^x \ln 2 \to$

$$Y' = \frac{2x - 2^x \ln 2}{\frac{1}{\ln 3} \cdot \frac{1}{Y} - 3Y^2}$$

8.) $\int f(x)\,dx = \sin(7x) + c \to D(\sin(7x) + c) = f(x)$

$\to 7\cos(7x) = f(x) \to f'(x) = -7^2 \sin(7x) \to$

$f''(x) = -7^3 \cos(7x)$

9.) $A = P(1 + \frac{r}{n})^{nt}$, $A = Pe^{rt} \to$

acct. AA $\to A = 500(1 + \frac{0.07}{12})^{12t}$, $A = 400e^{0.08t} \leftarrow$ acct. BB

a.) $t = 10$ yrs. $\to$ AA has \$1004.83 and BB has \$890.22, difference: \$<u>114.61</u>

b.) $t = 50$ yrs. $\to$ AA has \$16,390.21 and BB has \$21,839.26, difference: \$<u>5449.05</u>

10.) a.) $\int x^5(x^2 - 1)\,dx = \int (x^7 - x^5)\,dx$

$= \frac{1}{8}x^8 - \frac{1}{6}x^6 + c$

b.) $\int x(x^2-1)^5 dx$ (Let $u = x^2-1 \rightarrow du = 2x\,dx$

$= \frac{1}{2}\int u^5\, du$ $\rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\cdot\frac{1}{6}u^6 + c = \frac{1}{12}(x^2-1)^6 + c$

c.) $\int (7x+1)^{3/2} dx$ (Let $u = 7x+1 \rightarrow du = 7\,dx$

$= \frac{1}{7}\int u^{3/2} du$ $\rightarrow \frac{1}{7}du = dx$)

$= \frac{1}{7}\cdot\frac{1}{\frac{5}{2}}u^{5/2} + c = \frac{2}{35}(7x+1)^{5/2} + c$

d.) $\int \frac{e^x}{\sqrt{4+e^x}} dx$ (Let $u = 4+e^x \rightarrow$ $du = e^x dx$)

$= \int \frac{1}{\sqrt{u}} du = \int u^{-1/2} du = \frac{1}{\frac{1}{2}} u^{1/2} + c = 2(4+e^x)^{1/2} + c$

Extra Credit

1.) $Y = 2\ln(3X+1) - \ln(2-X)^2 \longrightarrow$ (switch)

$X = 2\ln(3Y+1) - \ln(2-Y)^2 \longrightarrow$

$X = \ln(3Y+1)^2 - \ln(2-Y)^2 \rightarrow X = \ln\frac{(3Y+1)^2}{(2-Y)^2} \rightarrow$

$X = \ln\left(\frac{3Y+1}{2-Y}\right)^2 \rightarrow \left(\frac{3Y+1}{2-Y}\right)^2 = e^X \rightarrow \frac{3Y+1}{2-Y} = e^{X/2} \rightarrow$

$3Y+1 = 2e^{X/2} - Ye^{X/2} \rightarrow 3Y + Ye^{X/2} = -1 + 2e^{X/2} \rightarrow$

$Y(3+e^{X/2}) = -1+2e^{X/2} \rightarrow Y = \frac{-1+2e^{X/2}}{3+e^{X/2}}$ (inverse)

2.) assume $A = Ce^{kt} \rightarrow A = 2358e^{kt}$ and

$t=10, A=4000 \rightarrow 4000 = 2358e^{10k} \rightarrow$

$k = \frac{1}{10}\ln\frac{4000}{2358} \rightarrow \boxed{A = 2358e^{\frac{1}{10}\ln\frac{4000}{2358}\cdot t}}$;

100 yrs. ago: let $t = -100$! $\rightarrow A \approx 12$ people!