Math 16B
Kouba
Exam 2

Please PRINT your name here : KEY

Your Exam ID Number ____________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO TAKE AN EXAM FOR ANOTHER PERSON. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive little or no credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 5 pages, including the cover page.

6. Include units on answers where units are appropriate.

7. You will be graded on proper use of integral and derivative notation.

8. You have until 2:00 p.m. sharp to finish the exam.

9. You may use the following trig identities :

a.) $\sin^2\theta + \cos^2\theta = 1$
b.) $1 + \tan^2\theta = \sec^2\theta$
c.) $\sin 2\theta = 2\sin\theta\cos\theta$
d.) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

1.) (7 pts. each) Determine the following indefinite integrals.

a.) $\int e^x(1+e^x)^8 \, dx$ $\quad$ (Let $u = 1+e^x \xrightarrow{D} du = e^x \, dx$)

$= \int u^8 \, du = \frac{1}{9}u^9 + C = \frac{1}{9}(1+e^x)^9 + C$

b.) $\int \frac{1}{x \ln x} \, dx$ $\quad$ (Let $u = \ln x \xrightarrow{D} du = \frac{1}{x} dx$)

$= \int \frac{1}{u} \, du = \ln|u| + C = \ln|\ln x| + C$

c.) $\int \frac{x+2}{3-x} \, dx$ $\quad$ (Let $u = 3-x \xrightarrow{D} du = -dx \rightarrow -du = dx$)

and $x = 3-u$

$= -\int \frac{(3-u)+2}{u} \, du = -\int \frac{5-u}{u} \, du = -\int \left(\frac{5}{u} - \frac{u}{u}\right) du$

$= -\int \left(5 \cdot \frac{1}{u} - 1\right) du = -(5 \cdot \ln|u| - u) + C$

$= -(5\ln|3-x| - (3-x)) + C$

d.) $\int xe^{2x} \, dx$ $\quad$ (Let $u = x$, $dv = e^{2x} dx$

$\rightarrow du = dx$, $v = \frac{1}{2}e^{2x}$)

$= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x} dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{2} \cdot \frac{1}{2}e^{2x} + C$

e.) $\int \sec^2 x \ \tan^2 x \ dx$ $\quad$ (Let $u = \tan x \xrightarrow{D} du = \sec^2 x \, dx$)

$$= \int u^2 \, du = \frac{1}{3} u^3 + C = \frac{1}{3} (\tan x)^3 + C$$

f.) $\int \tan^2 x \ dx = \int (\sec^2 x - 1) \, dx$

$$= \tan x - x + C$$

2.) (7 pts.) Compute T_4, the Trapezoidal Estimate using $n = 4$, for $\int_{-2}^{2} \sqrt{x+7} \ dx$.

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

$f(x) = \sqrt{x+7}$, $\quad h = \frac{2-(-2)}{4} = 1$

$$T_4 = \frac{h}{2} \left[f(-2) + 2f(-1) + 2f(0) + 2f(1) + f(2) \right]$$

$$= \frac{1}{2} \left[\sqrt{5} + 2\sqrt{6} + 2\sqrt{7} + 2\sqrt{8} + \sqrt{9} \right] \approx 10.54$$

3.) (7 pts.) Water is leaking from your bathroom faucet at the rate of $\sqrt{t+4}$ gallons per hour at time t hours. Determine the total amount of water leakage for $0 \leq t \leq 5$.

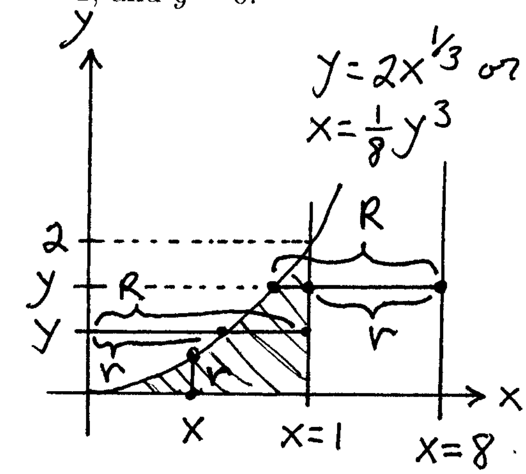
$$\text{TOTAL} = \int_0^5 \underbrace{\sqrt{t+4}}_{\text{gal./hr.}} \ \underbrace{dt}_{\text{hr.}}$$

$$= \frac{2}{3}(t+4)^{3/2} \Big|_0^5 = \frac{2}{3}(9)^{3/2} - \frac{2}{3}(4)^{3/2}$$

$$= \frac{2}{3}(27) - \frac{2}{3}(8) = \frac{38}{3} \text{ gal.} \approx 12.67 \text{ gal.}$$

4.) Consider the region bounded by the graphs of $y = 2 \cdot x^{1/3}$, $x = 1$, and $y = 0$.

a.) (8 pts.) Compute the AREA of the region.



$$\text{Area} = \int_0^1 2 \cdot x^{1/3}\,dx$$
$$= 2 \cdot \frac{3}{4} x^{4/3} \Big|_0^1$$
$$= \frac{3}{2}(1)^{4/3} - \frac{3}{2}(0)^{4/3}$$
$$= \frac{3}{2}(1) - \frac{3}{2}(0)$$
$$= \frac{3}{2}$$

b.) (7 pts.) Compute the VOLUME of the solid formed by revolving the region about the x-axis (SET UP BUT DO NOT EVALUATE THE INTEGRAL(S).).

$$\text{Vol} = \pi \int_0^1 (2x^{1/3})^2\,dx$$

c.) (7 pts.) Compute the VOLUME of the solid formed by revolving the region about the y-axis (SET UP BUT DO NOT EVALUATE THE INTEGRAL(S).).

$$\text{Vol} = \pi \int_0^2 R^2\,dy - \pi \int_0^2 r^2\,dy$$
$$= \pi \int_0^2 (1)^2\,dy - \pi \int_0^2 \left(\tfrac{1}{8}y^3\right)^2 dy$$

d.) (7 pts.) Compute the VOLUME of the solid formed by revolving the region about the line $x = 8$ (SET UP BUT DO NOT EVALUATE THE INTEGRAL(S).).

$$\text{Vol} = \pi \int_0^2 R^2\,dy - \pi \int_0^2 r^2\,dy$$
$$= \pi \int_0^2 \left(8 - \tfrac{1}{8}y^3\right)^2 dy - \pi \int_0^2 (7)^2\,dy$$

5.) (8 pts.) The temperature (^{o} F) of a rootbeer popsicle is given by $T = \dfrac{30t}{t^2+1}$ at time t hours. Find the average temperature of your popsicle for $0 \leq t \leq 2$.

$$AVE = \frac{1}{2-0}\int_0^2 \frac{30t}{t^2+1}\,dt = \frac{1}{2}\cdot 30 \cdot \frac{1}{2}\ln|t^2+1|\Big|_0^2$$

$$= \frac{15}{2}\ln 5 - \frac{15}{2}\cancel{\ln 1}^{0}$$

$$= \frac{15}{2}\ln 5 \approx 12.07\ ^\circ F$$

6.) (7 pts.) Compute the AREA of the region bounded by the graphs of $x = y^2$ and $x = 2 - y$. (SET UP BUT DO NOT EVALUATE INTEGRAL(S).)

$y^2 = 2 - y \rightarrow y^2 + y - 2 = 0 \rightarrow$

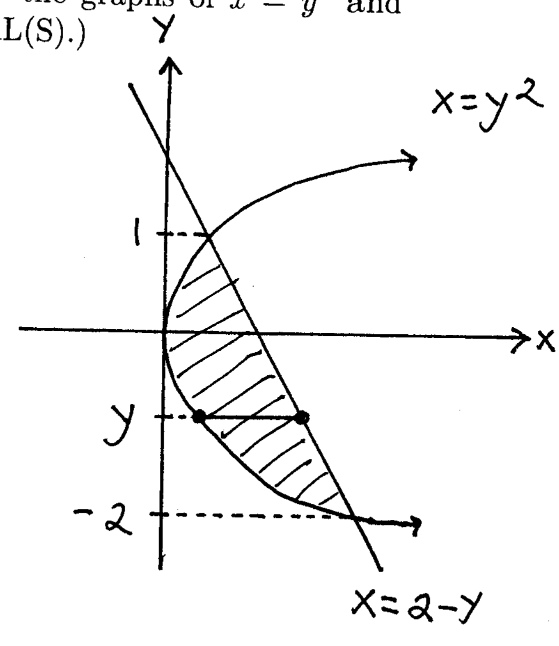
$(y-1)(y+2) = 0$

$\downarrow \qquad \downarrow$

$y = 1 \qquad y = -2$

$$AREA = \int_{-2}^{1} [(2-y) - y^2]\,dy$$

$\uparrow$ right $\quad \uparrow$ left



EXTRA CREDIT PROBLEM– The following problem is worth 10 points. This problem is OPTIONAL.

1.) $\displaystyle\int \frac{e^{-x}-1}{xe^{-x}-1}\,dx = \int \frac{e^{-x}-1}{xe^{-x}-1}\cdot\frac{e^x}{e^x}\,dx$

$$= \int \frac{1-e^x}{x-e^x}\,dx \quad (\text{Let } u = x - e^x \xrightarrow{D} du = (1-e^x)\,dx)$$

$$= \int \frac{1}{u}\,du = \ln|u| + C = \ln|x-e^x| + C$$