Math 16B
Exam 2 Solutions

1.) a.) $\int (x+1)e^{x^2+2x-5}\,dx$ (Let $u = x^2+2x-5 \rightarrow$

$du = (2x+2)\,dx \rightarrow \frac{1}{2}du = (x+1)\,dx$)

$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2+2x-5} + c$

b.) $\int_0^1 x(1-x)^{20}\,dx$ (Let $u = 1-x \rightarrow du = -dx \rightarrow$
$\hookrightarrow x = 1-u$

$-du = dx$ and $x: 0 \rightarrow 1$ so $u: 1 \rightarrow 0$)

$= -\int_1^0 (1-u)u^{20}\,du = -\int_1^0 (u^{20}-u^{21})\,du$

$= -\frac{u^{21}}{21} + \frac{u^{22}}{22}\Big|_1^0 = 0 - \left(\frac{-1}{21}+\frac{1}{22}\right) = \frac{1}{462}$

c.) $\int \sec^{10}x \cdot \tan x\,dx = \int \sec^9 x \cdot \sec x \tan x\,dx$

(Let $u = \sec x \rightarrow du = \sec x \tan x\,dx$)

$= \int u^9\,du = \frac{1}{10}u^{10} + c = \frac{1}{10}\sec^{10}x + c$

d.) $\int \frac{x^3+10}{x+3}\,dx$

$$\begin{array}{r}
x^2 - 3x + 9 \\
x+3 \,\overline{)\, x^3 + 10} \\
\underline{x^3 + 3x^2} \\
-3x^2 + 10 \\
\underline{-3x^2 - 9x} \\
9x + 10 \\
\underline{9x + 27} \\
-17
\end{array}$$

$= \int \left[x^2 - 3x + 9 + \frac{-17}{x+3}\right]dx$

$= \frac{x^3}{3} - \frac{3}{2}x^2 + 9x - 17\ln|x+3| + c$

e.) $\int \ln x\,dx$ (Let $u = \ln x$ $\quad dv = dx$
$du = \frac{1}{x}dx$ $\quad v = x$)

$= x\ln x - \int 1\,dx = x\ln x - x + c$

f.) $\int_0^4 |6-2x|\,dx = \int_0^3 (6-2x)\,dx + \int_3^4 -(6-2x)\,dx$

$= (6x-x^2)\Big|_0^3 + (-6x+x^2)\Big|_3^4$

$= (18-9) + (-24+16) - (-18+9) = 10$

g.) $\int (1+\sin 3x)^2\,dx = \int (1 + 2\sin 3x + \sin^2 3x)\,dx$

$(\sin^2\theta = \frac{1}{2}(1-\cos 2\theta))$

$= \int \left(1 + 2\cdot\sin 3x + \frac{1}{2}(1-\cos 6x)\right) dx$

$= x + 2\cdot\frac{-1}{3}\cos 3x + \frac{1}{2}\cdot\left(x - \frac{1}{6}\sin 6x\right) + C$

$= \frac{3}{2}x - \frac{2}{3}\cos 3x - \frac{1}{12}\sin 6x + C$

h.) $\int x\cos x\,dx$ (Let $u = x$, $dv = \cos x\,dx$
$du = 1\,dx$, $v = \sin x$)

$= x\sin x - \int \sin x\,dx$

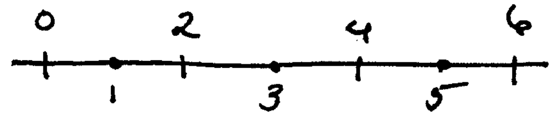
$= x\sin x - (-\cos x) + C$

i.) $\int e^{2x}\sqrt{3+e^x}\,dx = \int e^x\cdot e^x\sqrt{3+e^x}\,dx$

(Let $u = 3+e^x \rightarrow du = e^x\,dx$ and $e^x = u-3$)

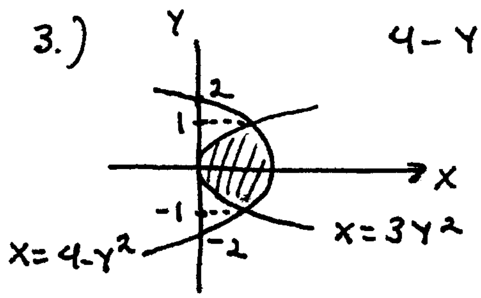
$= \int (u-3)\sqrt{u}\,du = \int (u^{3/2} - 3u^{1/2})\,du$

$= \frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{5}(3+e^x)^{5/2} - 2(3+e^x)^{3/2} + C$

2.) 

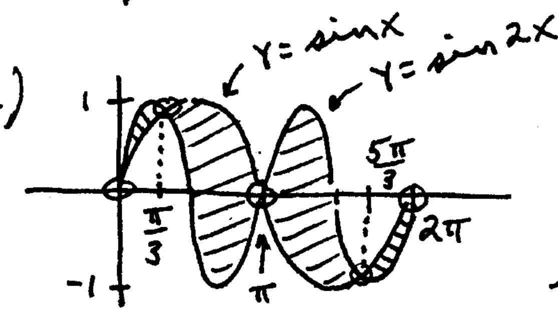
$f(x) = \sqrt{4+x^2}$ so

$$M_3 = \frac{6-0}{3}\left[f(1) + f(3) + f(5)\right]$$
$$= 2\left[\sqrt{5} + \sqrt{13} + \sqrt{29}\right] \approx 22.454$$

3.) 

$4 - Y^2 = 3Y^2 \rightarrow 4Y^2 = 4 \rightarrow Y = \pm 1$ so

$$\text{AREA} = \int_{-1}^{1}\left[(4-Y^2) - (3Y^2)\right]dy$$

4.) 

$\sin 2x = \sin x \rightarrow$
$2\sin x\cos x - \sin x = 0 \rightarrow$
$\sin x\,(2\cos x - 1) = 0 \rightarrow$
$\sin x = 0 \rightarrow x = 0, \pi, 2\pi$ or
$\cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

$$\text{AREA} = \int_0^{\frac{\pi}{3}}(\sin 2x - \sin x)\,dx + \int_{\frac{\pi}{3}}^{\pi}(\sin x - \sin 2x)\,dx$$
$$+ \int_{\pi}^{\frac{5\pi}{3}}(\sin 2x - \sin x)\,dx + \int_{\frac{5\pi}{3}}^{2\pi}(\sin x - \sin 2x)\,dx$$

5.) $V = 50te^{-t^2} \rightarrow$ distance $L = \int 50te^{-t^2}dt \rightarrow$
$L = -25e^{-t^2} + C$ so total distance is
$L(2) - L(0) = (-25e^{-4} + C) - (-25e^{0} + C) \approx 24.5$ mi.

6.) $T = 40 + \frac{50}{(t+1)^2}$

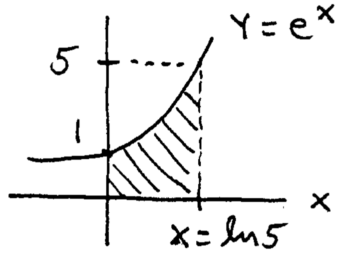
a.) $T(0) = 90°F$, $T(4) = 42°F$

b.) $\frac{dT}{dt} = \frac{-100}{(t+1)^3}$ so at $t = 4$, $\frac{dT}{dt} = {}^-0.8\,{}^\circ F/hr.$

c.) $AVE = \frac{1}{4-0}\int_0^4 \left(40 + \frac{50}{(t+1)^2}\right) dt$

$= \frac{1}{4}\left(40t - \frac{50}{t+1}\right)\Big|_0^4 = \frac{1}{4}(160 - 10) - \frac{1}{4}(0 - 50) = 50^\circ F$

7.)



$Y = e^x$ or $x = \ln Y$

a.) $VOL = \pi \int_0^{\ln 5} (e^x)^2\, dx$

b.) $Vol = \pi \int_0^1 (\ln 5)^2\, dy$

$+ \pi \int_1^5 (\ln 5)^2\, dy - \pi \int_1^5 (\ln Y)^2\, dy$

<u>Extra Credit</u>:

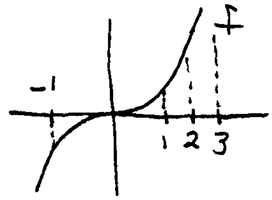
1.) $\int \sqrt{1+\sqrt{x}}\, dx$ (Let $u = 1+\sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}}\, dx \rightarrow$
$\hookrightarrow \sqrt{x} = u - 1$

$du = \frac{1}{2(u-1)}\, dx \rightarrow 2(u-1)\, du = dx$)

$= \int \sqrt{u} \cdot 2(u-1)\, du = 2\int \left(u^{3/2} - u^{1/2}\right) du$

$= 2\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C = \frac{4}{5}(1+\sqrt{x})^{5/2} - \frac{4}{3}(1+\sqrt{x})^{3/2} + C$

2.)



$4 = \int_{-1}^2 f(x)\, dx = \underbrace{\cancel{\int_{-1}^1 f(x)\, dx}}_{0,\text{ since } f \text{ odd}} + \int_1^2 f(x)\, dx$

or $\int_1^2 f(x)\, dx = 4$ and

$7 = \int_1^3 f(x)\, dx = \underbrace{\cancel{\int_1^2 f(x)\, dx}}_{4} + \int_2^3 f(x)\, dx$ so

$\int_2^3 f(x)\, dx = 7 - 4 = 3$