Practice Exam 3 Solutions

1.) a.) $\int \frac{3\cos x}{(7+\sin x)^4}\,dx$ (Let $u = 7+\sin x \rightarrow$ $du = \cos x\,dx$)

$= \int \frac{3}{u^4}\,du = 3\int u^{-4}\,du = 3\cdot\frac{u^{-3}}{-3} + C = -(7+\sin x)^{-3} + C$

b.) $\int x^2 e^{-x}\,dx$ (Let $u = x^2$, $dv = e^{-x}\,dx$ $\rightarrow du = 2x\,dx$, $v = -e^{-x}$)

$= -x^2e^{-x} - -2\int xe^{-x}\,dx$ (Let $u = x$, $dv = e^{-x}\,dx$ $\rightarrow du = dx$, $v = -e^{-x}$)

$= -x^2e^{-x} + 2\left[-xe^{-x} - -\int e^{-x}\,dx\right]$

$= -x^2e^{-x} - 2xe^{-x} + 2(-e^{-x}) + C$

c.) $\int e^x e^x (5+e^x)^6\,dx$ (Let $u = 5+e^x$, $du = e^x\,dx$ and $e^x = u-5$)

$= \int (u-5)u^6\,du = \int (u^7 - 5u^6)\,du = \frac{1}{8}u^8 - \frac{5}{7}u^7 + C$

$= \frac{1}{8}(5+e^x)^8 - \frac{5}{7}(5+e^x)^7 + C$

d.) $\int (\cos 2x + \cos^2 2x)\,dx$ $\left(\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)\right)$

$= \int \left(\cos 2x + \frac{1}{2}(1+\cos 4x)\right)dx$

$= \frac{1}{2}\sin 2x + \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right) + C$

[redacted]

2.) a.) $\int \frac{3x+5}{(x-2)(x+2)}\,dx = \int\left(\frac{A}{x-2} + \frac{B}{x+2}\right)dx$

($3x+5 = A(x+2) + B(x-2) \rightarrow$ let $x=2$: $11 = 4A \rightarrow$ $A = \frac{11}{4}$, let $x=-2$: $-1 = -4B \rightarrow B = \frac{1}{4}$)

$$= \int \left(\frac{\frac{11}{4}}{x-2} + \frac{\frac{1}{4}}{x+2} \right) dx = \frac{11}{4} \ln|x-2| + \frac{1}{4} \ln|x+2| + c$$

b.) $\int \frac{x-4}{x^2(x+1)}\, dx = \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \right) dx$

$(x-4 = Ax(x+1) + B(x+1) + Cx^2 \to$

let $x=0: \ -4 = B$

let $x=-1: \ -5 = C$

let $x=1: \ -3 = 2A - 8 - 5 \to A = 5)$

$$= \int \left(\frac{5}{x} + \frac{-4}{x^2} + \frac{-5}{x+1} \right) dx = 5 \ln|x| + \frac{4}{x} - 5 \ln|x+1| + c$$

c.) $\int (\sec^2 3x - 2 \sec 3x \tan 3x + \tan^2 3x)\, dx$

$$= \int (\sec^2 3x - 2 \sec 3x \tan 3x + (\sec^2 3x - 1))\, dx$$

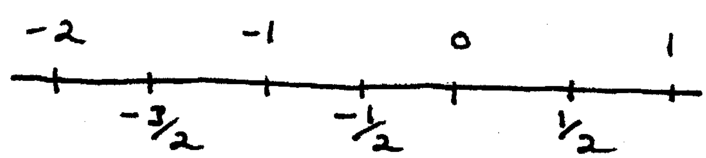
$$= \frac{1}{3} \tan 3x - 2 \cdot \frac{1}{3} \cdot \sec 3x + \frac{1}{3} \tan 3x - x + c$$

d.) $\int \cos^3 x\, dx = \int \cos x \cdot \cos^2 x\, dx$

$= \int \cos x\, (1 - \sin^2 x)\, dx$ (Let $u = \sin x \to du = \cos x\, dx$)

$$= \int (1-u^2)\, du = u - \frac{1}{3}u^3 + c = \sin x - \frac{1}{3}\sin^3 x + c$$

3.) $f(x) = \sqrt{3+4x^2}$



$$S_6 = \frac{1-(-2)}{3(6)} \left[f(-2) + 4f\left(\tfrac{-3}{2}\right) + 2f(-1) + 4f\left(\tfrac{-1}{2}\right) + 2f(0) + 4f\left(\tfrac{1}{2}\right) + f(1) \right]$$

$$= \frac{1}{6} \left[\sqrt{19} + 4\sqrt{12} + 2\sqrt{7} + 4\sqrt{4} + 2\sqrt{3} + 4\sqrt{4} + \sqrt{7} \right]$$

≈ 7.603

4.) $f(x) = \ln(1+3x) \to f'(x) = 3(1+3x)^{-1} \to$
$f''(x) = -3(1+3x)^{-2}(3) = \dfrac{-9}{(1+3x)^2}$ so

$\max\limits_{0 \le x \le 2} |f''(x)| = \max\limits_{0 \le x \le 2} \left|\dfrac{-9}{(1+3x)^2}\right| = 9$ then

$|E_n| \le \dfrac{(2-0)^3}{12n^2}(9) \le 0.00003 \to \dfrac{(8)(9)}{(12)(0.00003)} \le n^2 \to$

$n \ge 447.2$ so choose $n = 448$

5.) a.) $\int_0^\infty x^2 e^{-x^3} dx = \lim\limits_{A \to \infty} \int_0^A x^2 e^{-x^3} dx$

$= \lim\limits_{A \to \infty} \frac{-1}{3} e^{-x^3} \Big|_0^A = \lim\limits_{A \to \infty} \left(\frac{-1}{3} \underbrace{e^{-A^3}}_{0} - \frac{-1}{3} e^0\right) = \frac{1}{3}$

b.) $\int_0^3 \frac{x}{9-x^2} dx = \lim\limits_{A \to 3^-} \int_0^A \frac{x}{9-x^2} dx$

$= \lim\limits_{A \to 3^-} \frac{-1}{2} \ln|9-x^2| \Big|_0^A = \lim\limits_{A \to 3^-} \left(\frac{-1}{2}\ln|9-A^2| - \frac{-1}{2}\ln 9\right)$

$= \left(\frac{-1}{2}\right)(-\infty) + \frac{1}{2}\ln 9 = +\infty$ (Diverges)

6.) $\int \sqrt{4x^2+16x}\, dx = 2\int \sqrt{(x^2+4x+4)-4}\, dx$

$= 2\int \sqrt{(x+2)^2 - 2^2}$ (Let $u = x+2$, $a = 2$)

$= 2 \cdot \frac{1}{2}\left[(x+2)\sqrt{(x+2)^2-2^2} - 2^2 \ln\left|(x+2) + \sqrt{(x+2)^2-2^2}\right|\right] + C$

EXTRA CREDIT: $\int \frac{1}{x\sqrt{x+1}} dx$ (Let $u = \sqrt{x+1} \to$ $x = u^2 - 1$

$du = \frac{1}{2\sqrt{x+1}} dx \to 2u\, du = dx$)

$= \int \frac{1}{(u^2-1)\cancel{u}} 2\cancel{u}\, du = \int \frac{2}{u^2-1} du = \int \left(\frac{A}{u-1} + \frac{B}{u+1}\right) du$

$$\left(2 = A(u+1) + B(u-1) \rightarrow \right.$$
$$\text{let } u=1:\ 2 = 2A \rightarrow A = 1$$
$$\left. \text{let } u=-1:\ 2 = -2B \rightarrow B = -1 \right)$$

$$= \int \left(\frac{1}{u-1} + \frac{-1}{u+1} \right) du$$

$$= \ln|u-1| - \ln|u+1| + C$$

$$= \ln\left|\sqrt{x+1} - 1\right| - \ln\left|\sqrt{x+1} + 1\right| + C$$