Math 16B
Final Exam Solutions

1.) a.) $\int \frac{e^{7x}}{50+e^{7x}}\,dx$ (Let $u = 50+e^{7x} \rightarrow$
$du = 7e^{7x}dx \rightarrow \frac{1}{7}du = e^{7x}dx$)
$= \frac{1}{7}\int \frac{1}{u}\,du = \frac{1}{7}\ln|u| + c = \frac{1}{7}\ln|50+e^{7x}| + c$

b.) $\int x\sqrt{5-x}\,dx$ (Let $u = 5-x \rightarrow du = -dx \rightarrow$
$-du = dx$ and $x = 5-u$)
$= -\int (5-u)\sqrt{u}\,du = -\int (5u^{1/2} - u^{3/2})\,du$
$= -(5\cdot\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}) + c = \frac{-10}{3}(5-x)^{3/2} + \frac{2}{5}(5-x)^{5/2} + c$

c.) $\int x\ln(x+1)\,dx$ (Let $u = \ln(x+1)$, $dv = x\,dx$
$du = \frac{1}{x+1}dx$, $v = \frac{x^2}{2}$)
$= \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\int \frac{x^2}{x+1}\,dx$
$= \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\int \left(x - 1 + \frac{1}{x+1}\right)dx$
$= \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\left(\frac{x^2}{2} - x + \ln|x+1|\right) + c$

$$\begin{array}{r} x-1 \\ x+1 \overline{)\, x^2 \qquad} \\ \underline{x^2+x} \\ -x \\ \underline{-x-1} \\ 1 \end{array}$$

d.) $\int (\cos x + \sec x)^2 dx = \int (\cos^2 x + 2 + \sec^2 x)\,dx$
$= \int \left(\frac{1}{2}(1+\cos 2x) + 2 + \sec^2 x\right)dx$
$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + 2x + \tan x + c$

e.) $\int \tan^3 x\,dx = \int \tan x \cdot \tan^2 x\,dx$
$= \int \tan x\,(\sec^2 x - 1)\,dx$
$= \int (\tan x \cdot \sec^2 x - \tan x)\,dx = \frac{1}{2}(\tan x)^2 - \ln|\sec x| + c$

f.) $\int (\ln x)^2 dx \quad \left(\text{Let } u = (\ln x)^2, \ dv = dx \right.$

$\left. du = \frac{2\ln x}{x}\,dx, \ v = x\right)$

$= x(\ln x)^2 - 2\int \ln x \, dx \quad \left(\text{Let } u = \ln x, \ dv = dx \right.$

$\left. du = \frac{1}{x}\,dx, \ v = x\right)$

$= x(\ln x)^2 - 2\left[x\ln x - \int 1\,dx\right]$

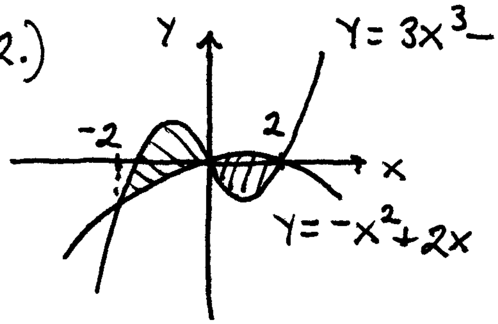
$= x(\ln x)^2 - 2x\ln x + 2x + C$

g.) $\int \frac{1+x}{x+3e^{-x}}\,dx = \int \frac{1+x}{x+3e^{-x}} \cdot \frac{e^x}{e^x}\,dx = \int \frac{e^x + xe^x}{xe^x+3}\,dx$

$\left(\text{Let } u = xe^x + 3 \rightarrow du = (xe^x + e^x)\,dx\right)$

$= \int \frac{1}{u}\,du = \ln|u| + C = \ln|xe^x+3| + C$

2.)



$3x^3 - x^2 - 10x = -x^2 + 2x \rightarrow$

$3x^3 - 12x = 0 \rightarrow$

$3x(x-2)(x+2) = 0$

$x=0 \quad x=2 \quad x=-2$

$\text{Area} = \int_{-2}^{0} (3x^3 - x^2 - 10x) - (-x^2 + 2x)\,dx$

$+ \int_{0}^{2} (-x^2 + 2x) - (3x^3 - x^2 - 10x)\,dx$

3.) $A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow 4000 = 1000\left(1 + \frac{0.0575}{12}\right)^{12t} \rightarrow$

$4 = (1.00479166667)^{12t} \rightarrow \ln 4 = 12t\ln(1.00479166667)$

$\rightarrow t = \frac{\ln 4}{12\ln(1.00479166667)} \approx 24.2 \text{ years}$

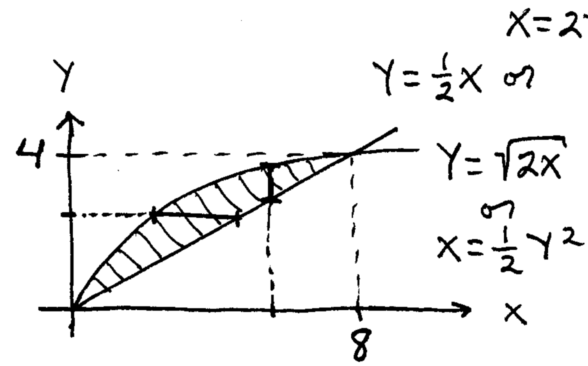
4.) $e^{xy} = y^3 + x^2 + \ln y \xrightarrow{D}$

$e^{xy}(xy' + y) = 3y^2y' + 2x + \frac{1}{y}y' \rightarrow$

$e^{xy}xy' + e^{xy}y = 3y^2y' + 2x + \frac{1}{y}y' \rightarrow$

$e^{xy}xy' - 3y^2y' - \frac{1}{y}y' = 2x - e^{xy}y \rightarrow$

$$y' = \frac{2x - e^{xy}y}{e^{xy}x - 3y^2 - 1/y} .$$

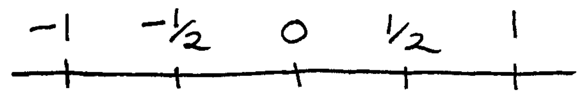


5.) a.)

$$Vol = \pi\int_0^8 (\sqrt{2x})^2\,dx - \pi\int_0^8 \left(\tfrac{1}{2}x\right)^2 dx$$

b.) $Vol = \pi\int_0^4 (2y)^2\,dy - \pi\int_0^4 \left(\tfrac{1}{2}y^2\right)^2 dy$

6.) $\int_{-1}^{1} \log_{10}(3+2x)\,dx$



$$T_4 = \frac{1-(-1)}{2(4)}\left[f(-1) + 2f(\tfrac{-1}{2}) + 2f(0) + 2f(\tfrac{1}{2}) + f(1)\right]$$

$$= \frac{1}{4}\left[\log 1 + 2\log 2 + 2\log 3 + 2\log 4 + \log 5\right]$$

≈ 0.865

7.) a.) $t=0 \rightarrow N = 100$ hogs, $t = 2 \rightarrow N = 420$ hogs, $\lim_{t\to\infty} N = 500$ hogs

b.) $N = 500 - 400(1+2t)^{-1} \rightarrow \frac{dN}{dt} = \frac{800}{(1+2t)^2}$

and $t = 2 \rightarrow N' = 32$ hogs/yr.

c.) $AVE = \frac{1}{2-0}\int_0^2 \left(500 - \frac{400}{1+2t}\right) dt$

$= \frac{1}{2}\left(500t - 200\ln(1+2t)\right)\Big|_0^2 = 500 - 100\ln 5$

≈ 339 hogs

8.) a.) $f(x)=\frac{1}{4\sqrt{x}} \geq 0$ for $1 \leq x \leq 9$ and

$\int_1^9 \frac{1}{4\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\Big|_1^9 = \frac{3}{2}-\frac{1}{2} = 1$ so f is a p.d.f.

b.) $\mu = E(x) = \int_1^9 x\left(\frac{1}{4\sqrt{x}}\right)dx = \int_1^9 \frac{1}{4}\sqrt{x}\,dx$

$= \frac{1}{4}\cdot\frac{2}{3}x^{3/2}\Big|_1^9 = \frac{1}{6}(9)^{3/2} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3} = 4\frac{1}{3}$ hrs.

c.) $\int_1^m \frac{1}{4\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\Big|_1^m = \frac{1}{2} \rightarrow \sqrt{m}-1=1 \rightarrow$

$\sqrt{m}=2 \rightarrow m=4$ hrs.

d.) $V(x) = \int_1^9 x^2\left(\frac{1}{4\sqrt{x}}\right)dx - \mu^2 = \int_1^9 \frac{x^{3/2}}{4}dx - \left(\frac{13}{3}\right)^2$

$= \frac{1}{4}\cdot\frac{2}{5}x^{5/2}\Big|_1^9 - \left(\frac{13}{3}\right)^2 = \frac{1}{10}(9)^{5/2} - \frac{1}{10} - \frac{169}{9}$

$= \frac{243}{10} - \frac{1}{10} - \frac{169}{9} \approx 5.42$ hrs.^{2}

e.) $\sigma = \sqrt{V(x)} \approx 2.33$ hrs.

f.) $P(x \geq 6) = \int_6^9 \frac{1}{4\sqrt{x}}\,dx = \frac{\sqrt{x}}{2}\Big|_6^9$

$= \frac{1}{2}\sqrt{9} - \frac{1}{2}\sqrt{6} \approx 0.28 = 28\%$

9.) Let x be net gain :

x	$P(x)$
$-\$5$	$264/300$
$+\$20$	$30/300$
$+\$45$	$5/300$
$+\$195$	$1/300$

$\mu = E(x) = (-5)\left(\frac{264}{300}\right) + (20)\left(\frac{30}{300}\right)$

$+ (45)\left(\frac{5}{300}\right) + (195)\left(\frac{1}{300}\right)$

$= \frac{-300}{300} = -\$1$

10.) $f(x) = \ln|x| - \ln(x^2+4) \rightarrow f'(x) = \frac{1}{x} - \frac{2x}{x^2+4}$

$= \frac{x^2+4-2x^2}{x(x^2+4)} = \frac{4-x^2}{x(x^2+4)} = 0$

f': $+ \quad 0 \quad - \quad | \quad + \quad 0 \quad -$

$x=-2 \qquad x=0 \qquad x=2$

$Y = \ln\frac{1}{4} \qquad Y = \ln\frac{1}{4}$

abs. max.

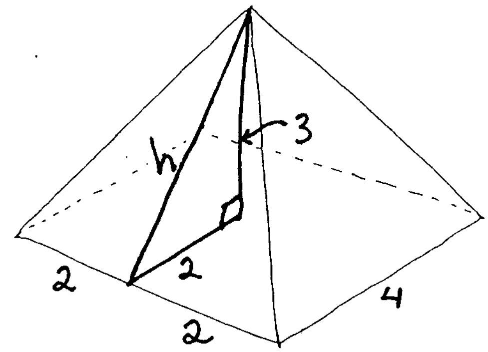
11.) $N = Ce^{kt}$ and $t = 1$ min., $N = 0.75C \rightarrow$
$0.75\cancel{C} = \cancel{C}e^{k} \rightarrow \ln 0.75 = \ln e^{k} = k \rightarrow$
$N = Ce^{t \ln 0.75}$; if $t = 10$ min. then
$N = 0.056C$ so 5.6% of original amount remains after 10 minutes

12.) acc. $a = -32 \rightarrow$ vel. $v = -32t + c$ $(t=0, v=0)$
$\rightarrow \boxed{v = -32t} \rightarrow$ ht. $s = -16t^2 + c$ $(t=0, s=15{,}000)$
$\rightarrow \boxed{s = -16t^2 + 15{,}000}$; if $s = 11{,}864$ then
$11{,}864 = -16t^2 + 15{,}000 \rightarrow 16t^2 = 3136 \rightarrow$
$t = 14$ sec., so

$$v = -448 \frac{\text{ft.}}{\text{sec.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \cdot \frac{1 \text{ mi.}}{5280 \text{ ft.}} \approx -305.5 \text{ mph}$$

Extra Credit:

$\boxed{\text{SA16}}$: a.) By Pythagorean Theorem



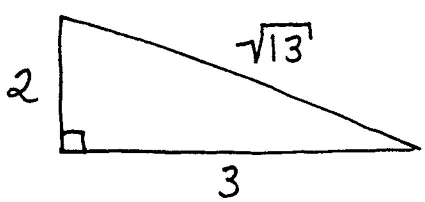
$2^2 + 3^2 = h^2 \rightarrow h = \sqrt{13}$,

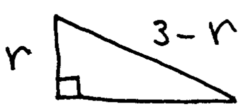
then area of triangle is $A = \frac{1}{2} bh = \frac{1}{2}(4)\sqrt{13} = 2\sqrt{13}$

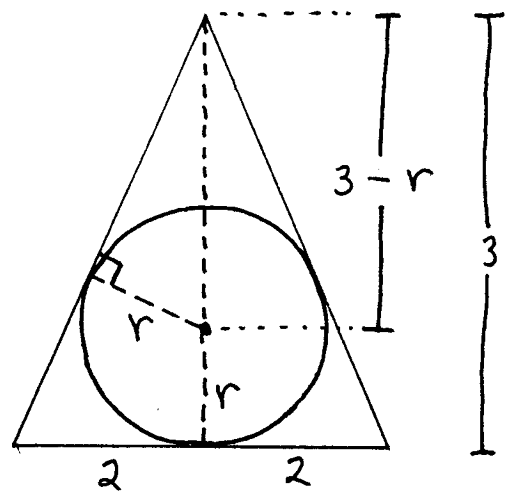
so total surface area of pyramid is

$(4)^2 + 4(2\sqrt{13}) = 16 + 8\sqrt{13}$;

b.) SIDE VIEW:







By similar triangles

$\frac{r}{3-r} = \frac{2}{\sqrt{13}} \rightarrow \sqrt{13}\, r = 6 - 2r \rightarrow$

$(\sqrt{13} + 2) r = 6 \rightarrow r = \frac{6}{\sqrt{13} + 2} \approx 1.07$

is radius of sphere