1. [7 points] Find and sketch the domain of $f(x, y) = \sqrt{x^2 + y^2 - 9}$ .

2. [10 points] Let $f(x, y) = \cos(1 + xy^2)$. Compute the following:
   (a) [4 points] $f_y$                    (b) [6 points] $f_{yx}$ .

3. [15 points] Let $f(x, y) = 5 + x^2 - x^2 y - y^2 - \frac{1}{3} y^3$ .

   (a) [8 points] Find all critical points f.
   (b) [7 points] Classify each critical point of f as a relative maximum, relative minimum, or saddle point .

4. [10 points] Use Newton's method with initial guess $x_1 = 0$ to compute two successive approximations to the solution of the equation $x^3 + 3x = 1$ .

5. [12 points] Determine whether the following sequences converge or diverge. Find the limit of the convergent ones.

   (a) $a_n = \frac{\sqrt{n^2 - 5}}{2n + 3}$                    (b) $a_n = (-1)^n \cos(\frac{1}{n})$

6. [18 points] Determine whether the following series converge or diverge. Clearly explain why.

   (a) $\sum\limits_{n=0}^{\infty} \frac{n}{500n + 79}$                    (b) $\sum\limits_{n=1}^{\infty} \frac{1}{n^2 \sqrt{n}}$

   (c) $\sum\limits_{n=1}^{\infty} (2n)! \, (\frac{2}{3})^n$

7. [12 points] Find the radius and interval of convergence of the power series
   $\sum\limits_{n=1}^{\infty} \frac{3^{-n}}{n + 1} (x + 1)^n$ . (Do not check end points.)

8. [10 points] Find the sum of the series $\sum\limits_{n=1}^{\infty} \frac{1}{3^n 2^{n-2}}$ .

9.   [26 points]   Evaluate the following double integrals.

(a) $\displaystyle\int_0^2 \int_0^{\sqrt{x}} y(x - y^2)^3 \, dy \, dx$        (b) $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 x \sin(1 + y^5) \, dy \, dx$

10.   [26 points]   Solve the following differential equations.

(a) $e^{x^2 - x} y' + y = 2xy$             (b) $xy' - 2y = x \ln x, \quad y(1) = 0$

11.   [12 points]   John is supposed to learn 1,000 French vocabulary words, of which he initially knows none. Suppose that he learns these words at a rate proportional to the number of words that he has not yet learned, and that he learns 150 words in the first 5 days. How many days does it take him to learn half the words? (Let $N$ be the number of words learned after $t$ days.)

12.   [13 points]   Use any method to find the 3rd–degree Taylor polynomial centered at $c = 1$ for $f(x) = \dfrac{1}{2x - 1}$.

13.   [13 points]   Approximate the definite integral $\displaystyle\int_0^1 x e^{-x^3} \, dx$ using a 7th–degree Taylor polynomial for $f(x) = x e^{-x^3}$. Express your answer as a fraction.

14.   [12 points]   Use the method of Lagrange multipliers to minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x - y + 2z = 3$ and $3x + y - z = 0$.