Math 17A
Kouba
Discussion Sheet 3

1.) Determine a formula (starting with $n = 0$) for each of the following sequences.

a.) $(1+1)^2, (1+1/2)^3, (1+1/3)^4, (1+1/4)^5, \cdots$ b.) $1, 0, 1, 4, 9, 16, \cdots$

c.) $0, 1/5, 2/6, 3/7, 4/8, 5/9, \cdots$ d.) $1, -8, 27, -64, 125, -216, \cdots$

e.) $3, 12, 27, 48, 75, 108, \cdots$ f.) $8, 10, 12, 14, 16, 18, \cdots$ g.) $5/2, 25/4, 125/6, 625/8, \cdots$

h.) $1/9, 1/3, 1, 3, 9, 27, \cdots$ i.) $1, 5, 10, 16, 23, 31, \cdots$ j.) $4, 2, 1, 1/2, 1/4, 1/8, \cdots$

k.) $0, 2, 6, 12, 20, 30, \cdots$ l.) $1, -5, 9, -13, 17, -21, \cdots$ m.) $1, -3, 1, -3, 1, -3, \cdots$

n.) $0, -1/5, 4/8, -9/11, 16/14, -25/17 \cdots$ o.) $0, 0, 0, 6, 24, 60, 120, 210, \cdots$

2.) Give a careful, step-by-step ϵ/N-proof that

a.) $\displaystyle\lim_{n\to\infty} \frac{2n+5}{1-2n} = -1$ b.) $\displaystyle\lim_{n\to\infty} \frac{n+100}{2n-50} = \frac{1}{2}$

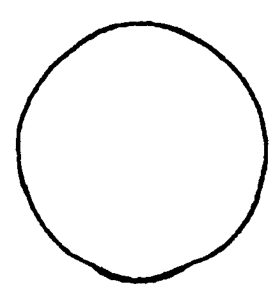
c.) $\displaystyle\lim_{n\to\infty} (0.08)^n = 0$ d.) $\displaystyle\lim_{n\to\infty} \frac{4}{\sqrt{3+\sqrt{n}}} = 0$ e.) $\displaystyle\lim_{n\to\infty} \frac{3^n}{3^n+10} = 1$

3.) Find all fixed points for each recursion.

a.) $a_{n+1} = \dfrac{6}{a_n - 5}$ b.) $a_{n+1} = \dfrac{70}{27 - 2a_n}$

c.) $a_{n+1} = \dfrac{2a_n^2}{8 - a_n^2}$ d.) $a_{n+1} = \sqrt{2 - a_n}$ e.) $a_{n+1} = \dfrac{a_n + 12}{a_n + 5}$

4.) What is the maximum number of distinct, non-overlapping parts into which a circle can be divided using 100 (non-parallel) lines ?



5.) Consider the Beverton-Holt Growth Recursion given by $N_{t+1} = \dfrac{54N_t}{30 + N_t}$ with $N_0 = 5$.

a.) Find all fixed points for this recursion.

b.) Identify the growth parameter R and the carrying capacity K.

c.) Plot the linear graph of the parent/offspring ratio N_t/N_{t+1} vs. the amount N_t.

d.) Use the recursion to determine the values of N_t for $t = 0, 1, 2, 3, \cdots, 10$.

e.) Plot the values you found in part d.).

6.) Use algebra to evaluate the following limits.

a.) $\lim_{x\to 1} \frac{x^2+3x-4}{x^2-x}$ b.) $\lim_{x\to -1} \frac{x^3+1}{x^2-1}$ c.) $\lim_{x\to 2} \frac{\sqrt{x+2}-x}{x-2}$

d.) $\lim_{x\to 1} \frac{x^6-1}{x^8-1}$ e.) $\lim_{x\to 7} \cos\frac{\pi}{2}x$ f.) $\lim_{x\to -1} \tan\frac{3\pi}{4}x$

g.) $\lim_{x\to\infty} \frac{3x-4}{3x+1000}$ h.) $\lim_{x\to\infty} \frac{2-x}{x^2+5}$ i.) $\lim_{x\to\infty} \frac{x^2-16}{x+16}$

j.) $\lim_{x\to 3^+} \frac{x+2}{x-3}$ k.) $\lim_{x\to 3^-} \frac{x+2}{x-3}$ l.) $\lim_{x\to 0^-} \frac{x-1}{x^2+x}$

m.) $\lim_{x\to\infty} (\sqrt{x+100}-\sqrt{x})$ n.) $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}}$ o.) $\lim_{x\to\infty} \frac{2^x-4^x}{3^x+4^x}$

7.) Assume that the weight N (in lbs.) at time t (in years) of a chimpanzee is given by the growth model $N = \frac{200e^t}{36+4e^t}$ for $t \geq 0$.

a.) What is the chimp's weight at birth ? at 1 year ? at 2 years ?

b.) When will the chimp reach a weight of 10 pounds ? 40 pounds ?

c.) What weight can we expect the chimp to reach as $t \to \infty$?

d.) Use a graphing calculator to sketch this growth equation.

8.) Determine all constants A so that the $\lim_{x\to -1} f(x)$ exists :

$$f(x) = \begin{cases} x^2 + A\ , & \text{if } x < -1 \\ x + A^2\ , & \text{if } x \geq -1 \end{cases}$$

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The following problem is for recreational purposes only.

9.) Without lifting your pencil, join all 16 dots with 6 straight lines.