Math 17A
Kouba
Discussion Sheet 4

1.) Evaluate the following limits.

a.) $\displaystyle\lim_{x\to\infty}\frac{4x^3-7x^2+2x-1}{5x^3+x^2-5x+1000}$

b.) $\displaystyle\lim_{x\to-\infty}\frac{x^3+1}{x^2+2}$

c.) $\displaystyle\lim_{x\to\infty}\frac{1-x}{x^3+x}$

d.) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{x^2-2x}}{x+2}$

e.) $\displaystyle\lim_{x\to\infty}(x-\sqrt{x^2+100}\ )$

f.) $\displaystyle\lim_{x\to\infty}\frac{e^{2x}+1}{e^{2x}+4}$

g.) $\displaystyle\lim_{x\to\infty}\frac{e^{x}+1}{e^{2x}+4}$

h.) $\displaystyle\lim_{x\to\infty}\frac{e^{2x}+1}{e^{x}+4}$

i.) $\displaystyle\lim_{x\to\infty}\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

j.) $\displaystyle\lim_{x\to-\infty}(e^{2x}+e^{-x})$

k.) $\displaystyle\lim_{x\to-\infty}\frac{e^{x}+1}{e^{x}+4}$

l.) $\displaystyle\lim_{x\to-\infty}\frac{e^{-2x}+1}{e^{-x}+4}$

m.) $\displaystyle\lim_{x\to-\infty}\frac{e^{-x}+1}{e^{-x}+4}$

n.) $\displaystyle\lim_{x\to\infty}\frac{e^{x}+e^{3x}}{e^{2x}+e^{3x}}$

o.) $\displaystyle\lim_{x\to-\infty}\frac{e^{x}+e^{3x}}{e^{2x}+e^{3x}}$

p.) $\displaystyle\lim_{x\to\infty}(e^{x}-\sqrt{e^{2x}+e^{x}}\ )$

2.) Determine if the following function is continuous at $x=0$:

$$g(x)=\begin{cases}\ln(x+e)\ , & \text{if } x>0\\ 1\ , & \text{if } x=0\\ \dfrac{e^x+e^{-x}}{2}\ , & \text{if } x<0\end{cases}$$

3.) Determine if the following function is continuous at $x=1$:

$$g(x)=\begin{cases}x^2-3x+4\ , & \text{if } x>1\\ 2\ , & \text{if } x=1\\ \dfrac{3x+1}{4}\ , & \text{if } x<1\end{cases}$$

4.) Determine if the following function is continuous at $x=2$:

$$g(x)=\begin{cases}\cos((\pi/4)x)\ , & \text{if } x\geq 2\\ \sin(3\pi x)\ , & \text{if } x<2\end{cases}$$

5.) Use shortcuts to determine the x-values for which each of the following functions is continuous. Briefly explain.

a.) $y = x^{1000} - \pi x + 1$
b.) $f(x) = (3x^2 + x - 2)\cos x$
c.) $f(x) = \dfrac{x}{x^2 - 4x - 5}$
d.) $f(x) = \ln x - e^x$
e.) $y = e^{\sin x}$
f.) $g(x) = \begin{cases} e^x \sin x\ , & \text{if } x \leq 0 \\ x^2 + x\ , & \text{if } x > 0 \end{cases}$

6.) Use limits and a "fake" graph to determine the values of constants A and B so that the following function is continuos for all values of x:

$$g(x) = \begin{cases} Ax^2 - Bx\ , & \text{if } x < -1 \\ 2 + 3x\ , & \text{if } -1 \leq x < 2 \\ \dfrac{Ax + B}{x - 1} + 7\ , & \text{if } x \geq 2 \end{cases}$$

7.) Use the Squeeze Principle (Sandwich Theorem) to evaluate the following limits or conclude that the limit does not exist.

a.) $\displaystyle\lim_{x\to\infty} \frac{x \sin x}{x^2 + 1}$ b.) $\displaystyle\lim_{x\to\infty} e^{-x} \cdot \cos 100x$ c.) $\displaystyle\lim_{x\to-\infty} \frac{e^{2x} \cdot \sin 5x}{2e^x + 1}$

d.) $\displaystyle\lim_{x\to 0} x^4 \sin(1/x)$ e.) $\displaystyle\lim_{x\to 0} (x^3 - 1)\cos(1/x)$

8.) Evaluate the following limits.

a.) $\displaystyle\lim_{x\to 0} \frac{\sin 3x}{5x}$ b.) $\displaystyle\lim_{x\to 0} \frac{\sin^2 4x}{x^2}$ c.) $\displaystyle\lim_{x\to 0} \frac{\tan x}{x^2 \cot x}$

d.) $\displaystyle\lim_{x\to 0} \frac{1 - \cos 3x}{x}$ e.) $\displaystyle\lim_{x\to \pi} \frac{\sin(5 \tan x)}{\tan x}$ f.) $\displaystyle\lim_{x\to 0} \frac{\sin 3x}{\sin x}$

9.) What can be determined about $\displaystyle\lim_{x\to\infty} f(x)$ for each of the following ?

a.) $\dfrac{x^2 - 5x}{x^2 + 10x} \leq f(x) \leq \dfrac{2x^2 + 50}{2x^2 + 100}$

b.) $\dfrac{3 - x}{x + 7} \leq f(x) \leq \dfrac{x + 4}{x - 4}$

c.) $x \sin(2/x) \leq f(x) \leq 1 + x \tan(1/x)$

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The following problem is for recreational purposes only.

10.) A snail is at the bottom of a well which is 100 feet deep. Each day it climbs up 5 feet and back down 4 feet. In how many days will the hapless snail reach the top of the well ?