Math 17A
Kouba
Discussion Sheet 8

1.) One hundred (100) years ago the gold mining town of Prospect, CA, had a population of 15,530 people. Today the population is 3750. Assuming exponential decay, what do you predict the population of Prospect will be 50 years from now ?

2.) Argon is a chemical element produced industrially by the fractional distillation of liquid air. Argon is mostly used as an inert shielding gas in welding and other high-temperature industrial processes where ordinarily non-reactive substances become reactive; for example, an argon atmosphere is used in graphite electric furnaces to prevent the graphite from burning. Argon gas also has uses in incandescent and fluorescent lighting, and other types of gas discharge tubes. Argon makes a distinctive blue-green gas laser. Argon is also used in fluorescent glow starters. The half-life of Argon is 1.827 hours. If you have a sample containing 500 milligrams of Argon, how many milligrams remain after 24 hours ?

3.) Assume that a fossilized bone found today contains 35% of it original amount of carbon-14. If the half-life of carbon-14 is 5730 years, estimate the age of the fossil.

4.) Use logarithmic differentiation to differentiate each function.

a.) $y = \dfrac{x^2}{(\tan x)^x}$ b.) $y = \sin^4 x \cdot \cos^5 x \cdot e^{5x} \cdot 4^{x^2} \cdot (3-x)^6$

c.) $y = \dfrac{x^4(x-3)^5 \sin x}{e^x \sqrt{x^2+x}}$ d.) $y = x^{x^x}$

5.) Let $f(x) = x^3 + x + 5$.

a.) Use a derivative to verify that f is one-to-one. Thus, $y = f(x)$ has an inverse function, f^{-1}.
b.) Note that $f(1) = 7$ and then find $D\{f^{-1}(7)\}$.

6.) Let $f(x) = \dfrac{x^3+1}{x^3-5}$.

a.) Verify algebraically that f is one-to-one, i.e., show that if $f(x_1) = f(x_2)$, then $x_1 = x_2$. Thus, $y = f(x)$ has an inverse function, f^{-1}.
b.) Note that $f(2) = 3$ and then find $D\{f^{-1}(3)\}$.

7.) Consider the exponential function $N(t) = Ce^{kt}$, where C and k are constants. Show that the rate at which $N(t)$ changes is directly proportional to $N(t)$.

8.) Use the fact that $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to compute each of the following limits (These problems are a bit tricky.).

a.) $\lim_{h \to 0} \dfrac{8^{1+h} - 8}{h}$ b.) $\lim_{h \to 0} \dfrac{5^{-2-h} - 5^{-2}}{h}$

9.) Use a linearization to estimate the value of

a.) $\sqrt{140}$ b.) $\sqrt{150}$ c.) $(29)^{1/3}$ d.) $e^{0.1}$

10.) The radius of a circle is measured with absolute percentage error of at most 3%. Use differentials to estimate the maximum absolute percentage error in computing the circle's

a.) circumference. b.) area.

(RECALL: For a circle : circumference $C = 2\pi r$ and area $A = \pi r^2$.)

11.) The radius of a sphere is measured with absolute percentage error of at most 4%. Use differentials to estimate the maximum absolute percentage error in computing the sphere's

a.) surface area. b.) volume.

(RECALL: For a sphere : surface area $S = 4\pi r^2$ and volume $V = (4/3)\pi r^3$.)

12.) Show that each function satisfies the assumptions of the Mean Value Theorem (MVT) over the given interval. Then determine all values of c guaranteed by the conclusion of the MVT.

a.) $f(x) = 3x^2 - 2x + 1$ on $[-1, 2]$ b.) $g(x) = \sqrt{9 - x^2}$ on $[-3, 0]$

c.) $f(x) = x^2(x-1)$ on $[-1, 3]$ d.) $f(x) = x + \dfrac{1}{x}$ on $[1, 4]$

e.) $f(x) = x + \tan(x/2)$ on $[0, \pi/3]$ f.) $f(x) = \begin{cases} 2x^3 + 1, & \text{if } 0 \le x \le 1 \\ 1 - x^2, & \text{if } -1 \le x < 0 \end{cases}$

g.) $f(x) = \sin x + \cos x$ on $[0, \pi]$ h.) $f(x) = \dfrac{x+1}{4-x}$ on $[0, 3]$

13.) Consider the function $f(x) = 1 - x^{2/3}$ on the interval $[-1, 8]$. Show that f does not satisfy all of the assumptions of the Mean Value Theorem.

14.) Solve $f'(x) = 0$ for x and set up a sign chart for f'. Determine points (x, y) for all relative maximums and minimums for f.

a.) $f(x) = x^3 - 6x^2$ b.) $y = x - 8\sqrt{x}$

c.) $y = \sin x - \cos x$ on interval $[0, 2\pi]$ d.) $y = x + \sin 2x$ on interval $[0, 2\pi]$

e.) $f(x) = \dfrac{x^2}{x-3}$ f.) $f(x) = \dfrac{2x}{x^2 + 9}$

g.) $y = xe^x$ h.) $y = x \ln x$ i.) $y = x(x-3)^4$ i.) $y = x^2(x-4)^2$

15.) The first derivative f' is given. Determine x-values for all relative maximums and minimums for f.

a.) $f'(x) = x^4(x-1)^2(x+3)^3$ b.) $f'(x) = \dfrac{x^2 - 3x - 10}{x^2 - 1}$

c.) $f'(x) = (e^x - 2)(\ln|x| - 1)$

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The following problem is for recreational purposes only.

16.) Find a hidden pattern and determine the next number in the sequence :

$0, 1, 3, 7, 14, 25, 41, 63, \cdots$