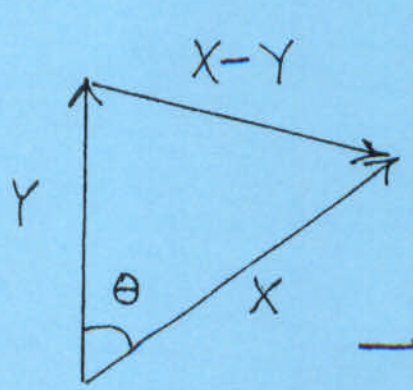


Math 17B

Kouba

An Alternate Formula for Dot Product

Let X and Y be vectors and consider the triangle formed by X , Y , and $X - Y$. Let θ be the angle between X and Y . By the Law of Cosines


$$\begin{aligned} |X-Y|^2 &= |X|^2 + |Y|^2 - 2|X||Y|\cos\theta \\ \rightarrow (X-Y) \cdot (X-Y) &= |X|^2 + |Y|^2 - 2|X||Y|\cos\theta \\ \rightarrow X \cdot X - X \cdot Y - Y \cdot X + Y \cdot Y &= |X|^2 + |Y|^2 - 2|X||Y|\cos\theta \\ \rightarrow \cancel{|X|^2} - X \cdot Y - X \cdot Y + \cancel{|Y|^2} &= \cancel{|X|^2} + \cancel{|Y|^2} - 2|X||Y|\cos\theta \\ \rightarrow -2(X \cdot Y) &= -2|X||Y|\cos\theta \end{aligned}$$

$$\rightarrow \boxed{X \cdot Y = |X||Y|\cos\theta}$$

or

$$\boxed{\cos\theta = \frac{X \cdot Y}{|X||Y|}}$$