

Math 17B

Kouba

An Example of Arc Length

Ex: Find the length of the graph of
 $y = \frac{x^2}{2} - \frac{1}{4} \ln x$ for $1 \leq x \leq 2$: Then

$$\underline{D} \rightarrow y' = \frac{1}{2} \cdot 2x - \frac{1}{4} \cdot \frac{1}{x} = x - \frac{1}{4x} \rightarrow$$

$$\text{Arc} = \int_1^2 \sqrt{1 + (y')^2} dx$$

$$= \int_1^2 \sqrt{1 + \left(x - \frac{1}{4x}\right)^2} dx$$

$$= \int_1^2 \sqrt{1 + x^2 - 2(x)\left(\frac{1}{4x}\right) + \frac{1}{16x^2}} dx$$

$$= \int_1^2 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}} dx$$

$$= \int_1^2 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}} dx$$

$$= \int_1^2 \sqrt{\frac{16x^4}{16x^2} + \frac{8x^2}{16x^2} + \frac{1}{16x^2}} dx$$

$$= \int_1^2 \sqrt{\frac{16x^4 + 8x^2 + 1}{16x^2}} dx$$

$$= \int_1^2 \frac{\sqrt{(4x^2 + 1)^2}}{\sqrt{16x^2}} dx$$

$$= \int_1^2 \frac{4x^2 + 1}{4x} dx$$

$$= \int_1^2 \left(\frac{4x^2}{4x} + \frac{1}{4x} \right) dx$$

$$= \int_1^2 \left(x + \frac{1}{4} \cdot \frac{1}{x} \right) dx$$

$$= \left(\frac{1}{2} x^2 + \frac{1}{4} \cdot \ln|x| \right) \Big|_1^2$$

$$= \left(2 + \frac{1}{4} \ln 2 \right) - \left(\frac{1}{2} + \frac{1}{4} \overset{0}{\ln 1} \right)$$

$$= \frac{3}{2} + \frac{1}{4} \ln 2$$