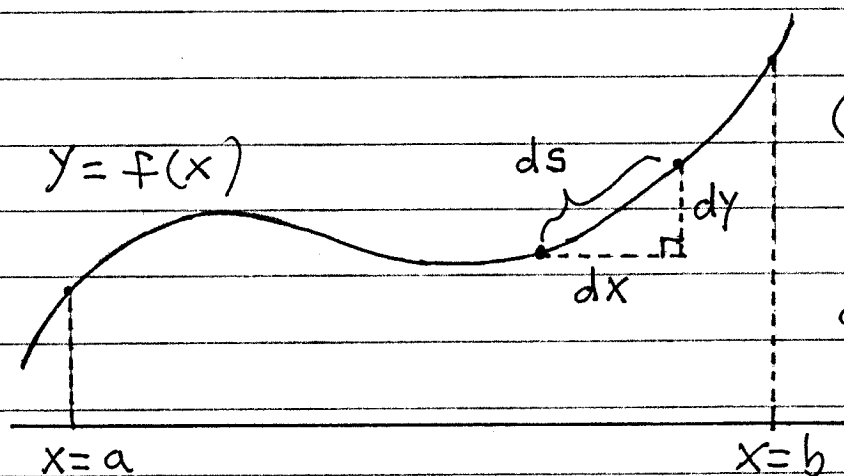


Math 17B
Kouba
Arc Length

Assume $y=f(x)$ is a continuous function from $x=a$ to $x=b$. We seek a formula for the length of its graph from $x=a$ to $x=b$.



Assume that
 $(ds)^2 = (dx)^2 + (dy)^2$
so that

$$ds = \sqrt{(dx)^2 + (dy)^2};$$

then

arc length from $x=a$ to $x=b$ is

$$ARC = \int_a^b ds = \int_a^b \sqrt{(dx)^2 + (dy)^2}$$

$$= \int_a^b \sqrt{\left(1 + \frac{(dy)^2}{(dx)^2}\right) (dx)^2}$$

$$= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx, \quad \text{i.e.,}$$

$$ARC = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$