

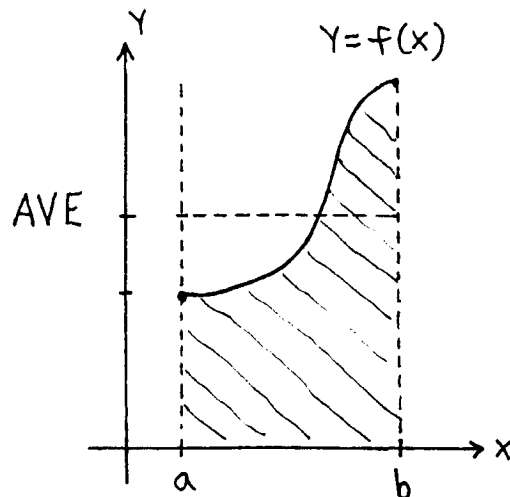
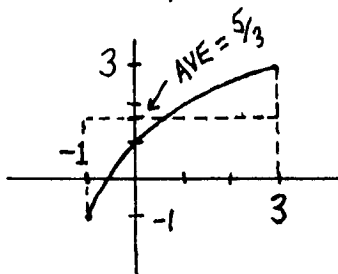
Math 17B
Kouba
Average Value of a Function

DEFINITION : The *average value*, AVE, of a function f on the interval $[a, b]$ is that y-value which determines the height of the rectangle (See diagram.) which has area exactly equal to the definite integral

$$\int_a^b f(x) \, dx, \text{ that is, } \text{AVE}(b-a) = \int_a^b f(x) \, dx, \text{ or } \text{AVE} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

EXAMPLE : Find the average value of $f(x) = -1 + 2\sqrt{x+1}$ on the interval $[-1, 3]$.

$$\begin{aligned} \text{AVE} &= \frac{1}{3-(-1)} \int_{-1}^3 (-1 + 2\sqrt{x+1}) \, dx \\ &= \frac{1}{4} \left[-x + 2 \cdot \frac{2}{3} (x+1)^{3/2} \right] \Big|_{-1}^3 \\ &= \frac{1}{4} \left(-3 + \frac{4}{3} (4)^{3/2} \right) - \frac{1}{4} \left(1 + \frac{4}{3} (0)^{3/2} \right) \\ &= \frac{1}{4} \left(-3 + \frac{4}{3} (8) \right) - \frac{1}{4} = \frac{20}{12} = \frac{5}{3} \end{aligned}$$



EXAMPLE : Money is withdrawn from an account in such a manner that the amount of money A in the account at time t years is given by the equation $A = 800,000 / (t + 2)^3$ for $t \geq 0$.

- What is the initial amount of money in the account ?
- How much money is in the account after 10 years ?
- What is the average amount of money in the account from $t = 0$ years to $t = 10$ years ?
- When is the average amount of money in the account equal to the average from part c.) ?