

Example : Solve $\frac{dN}{dt} = N - 1$:

$$\frac{dN}{dt} - N = -1 \rightarrow \frac{dN}{dt} + (-1)N = -1$$

$$\rightarrow \mu = e^{\int -1 dt} = e^{-t + c^0} = e^{-t} \rightarrow$$

$$e^{-t} \cdot \frac{dN}{dt} - e^{-t} \cdot N = -e^{-t} \rightarrow$$

$$D\{e^{-t}N\} = -e^{-t} \rightarrow$$

$$e^{-t}N = \int -e^{-t} dt \rightarrow$$

$$e^{-t}N = e^{-t} + C \rightarrow$$

$$N = \frac{e^{-t}}{e^{-t}} + \frac{C}{e^{-t}} \rightarrow$$

$$\boxed{N = 1 + Ce^t}$$

Note : If $\underline{N(0)=1}$, then $1 = 1 + Ce^0 = 1 + C$

$$\rightarrow C = 0 \rightarrow \boxed{N=1} ;$$

if $\underline{N(0)=2}$, then $2 = 1 + Ce^0 = 1 + C \rightarrow$

$$C = 1 \rightarrow \boxed{N = 1 + e^t} ;$$

if $\underline{N(0)=\frac{1}{2}}$, then $\frac{1}{2} = 1 + Ce^0 = 1 + C \rightarrow$

$$C = -\frac{1}{2} \rightarrow \boxed{N = 1 - \frac{1}{2}e^t}$$

CREATE A DIRECTION FIELD FOR A DIFFERENTIAL EQUATION

FIRST
FILL
OUT
A
TABLE
OF
VALUES
FOR

$t,$
 $N,$

AND

$\frac{dN}{dt}$

Consider D.E. $\frac{dN}{dt} = N - 1$

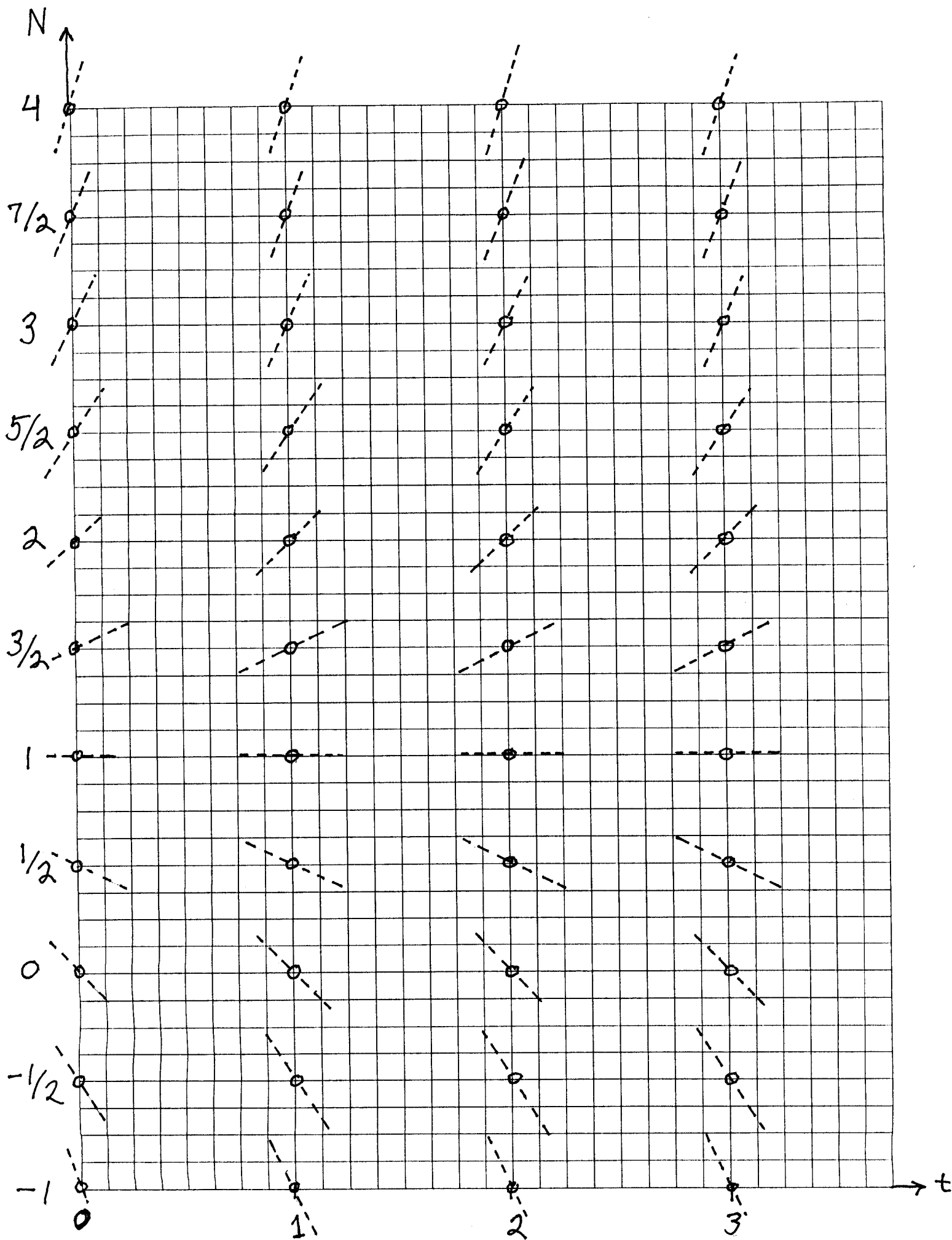
<u>t</u>	<u>N</u>	<u>$\frac{dN}{dt}$ (SLOPE of graph of N)</u>
0, 1, 2, 3	-1	-2
0, 1, 2, 3	-1/2	-3/2
0, 1, 2, 3	0	-1
0, 1, 2, 3	1/4	-3/4
0, 1, 2, 3	1/2	-1/2
0, 1, 2, 3	3/4	-1/4
0, 1, 2, 3	1	0
0, 1, 2, 3	5/4	1/4
0, 1, 2, 3	3/2	1/2
0, 1, 2, 3	7/4	3/4
0, 1, 2, 3	2	1
0, 1, 2, 3	5/2	3/2
0, 1, 2, 3	3	2
0, 1, 2, 3	7/2	5/2
0, 1, 2, 3	4	3

The values of N and $\frac{dN}{dt}$ are the same for $t=0, 1, 2, 3, 4, \dots$

NOW
DRAW
A
SLOPE
OF
VALUE

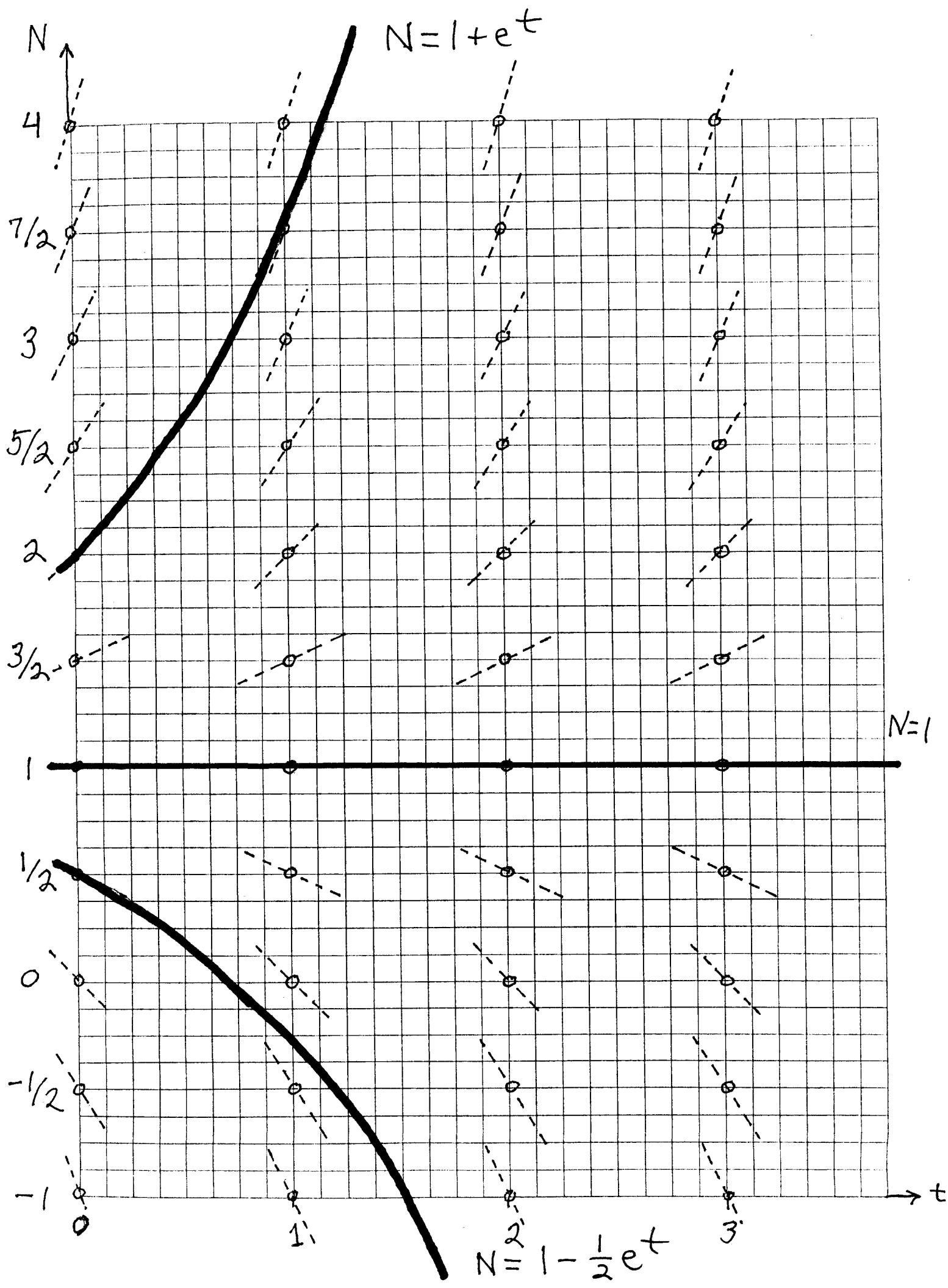
$$\frac{dN}{dt}$$

AT
EACH
POINT
(t, N)



NOW
START
AT
ANY
POINT
AND
FOLLOW
THE
DIRECTION
FIELD

LEFT TO RIGHT



Ta-da!