

Math 17B  
Kouba  
Discussion Sheet 1

1.) Determine the most general anti-derivative for each function by "guessing and checking."

- a.) 5      b.)  $x^2 - 3x + 5$       c.)  $(x + 1)(x^2 - 3)$       d.)  $\frac{x^3 - x + 7}{x^2}$
- e.)  $\sin 3x + 3 \sin x$       f.)  $\sec^2 7x + \sec 3x \tan 3x$       g.)  $\sec^2 x \cdot \tan^2 x$
- h.)  $(\sec x - \tan x)^2$       i.)  $e^{3x} - x^2 e^{x^3} + e$       j.)  $3^{x-2} + 5 \cdot 4^{3x}$
- k.)  $(1 + 2^x)(1 - 3^x)$       l.)  $\frac{1 + 2^x}{3^x}$       m.)  $x^2 \cos x + 2x \sin x$

2.) Evaluate the following sums.

- a.)  $\sum_{i=1}^n 9$       b.)  $\sum_{i=1}^{1053} 9$       c.)  $\sum_{i=34}^{867} 9$       d.)  $\sum_{i=1}^{50} i(2i + 3)$       e.)  $\sum_{i=1}^{60} (5i - i^2)$
- f.)  $\sum_{i=26}^{62} (5i - i^2)$       g.)  $\sum_{i=1}^{30} (\ln(i + 2) - \ln(i + 1))$       h.)  $\sum_{i=1}^4 \cos \pi i$       i.)  $\sum_{i=1}^{17} \cos \pi i$
- j.)  $\sum_{i=1}^n \cos \pi i$

3.) Prove that  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ .

4.) Evaluate the following sums.

- a.)  $1 + 2 + 3 + 4 + \cdots + 75$       b.)  $151 + 152 + 153 + \cdots + 364$
- c.)  $1 + 2 + 4 + 8 + 16 + \cdots + 1,048,576$       d.)  $1.1^6 + 1.1^7 + 1.1^8 + \cdots + 1.1^{200}$

5.) a.) Sketch the graph of  $y = 3x^2 + 2$  on the interval  $[0, 1]$ .

b.) Estimate the area of the region below the graph of  $y = 3x^2 + 2$  and above  $[0, 1]$  using rectangles above three equal subdivisions and

- i.) left endpoints of the subdivisions.  
ii.) right endpoints of the subdivisions.  
iii.) midpoints of the subdivisions.

6.) Consider the following sequence of numbers : 1, 5, 9, 13, 17, 21, 25,  $\dots$ . Find the sum of the 61st through the 127th numbers in this sequence.

7.) Consider the following sequence of numbers : 1, 2, 5, 10, 17, 26, 37, 50, 65,  $\dots$  . Find the sum of the first 200 numbers in this sequence.

8.) A Sooper Dooper Ball is thrown straight up from ground level and reaches a height of 100 feet before it falls back to earth and begins bouncing. On each bounce it reaches a height equal to 95% of the distance it fell.

a.) How high is the 6th bounce ? the 35th bounce ?

b.) What is the total distance that the ball travels through the air as it strikes the ground for the 50th time ?

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THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

9.) Use the fact that  $1 + r + r^2 + r^3 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}$  to derive a formula for  $1 + 2 \cdot r + 3 \cdot r^2 + 4 \cdot r^3 + \dots + (n + 1) \cdot r^n$ . What is the value of  $1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 31 \cdot 2^{30}$  ?