

Math 17B  
Kouba  
Discussion Sheet 3

1.) Use any method to determine the following indefinite integrals (antiderivatives).

a.)  $\int \frac{x}{\sqrt{1-x^2}} dx$    b.)  $\int \frac{e^{2x}}{1+e^{2x}} dx$    c.)  $\int \cos 5x dx$    d.)  $\int \frac{1+\sin x}{\cos^2 x} dx$

e.)  $\int (\sec x + \sec^2 x) dx$    f.)  $\int \tan^2 x \sec^2 x dx$    g.)  $\int \frac{(x+2)(x+3)}{x+1} dx$

h.)  $\int (x^2+1)(x^3+3x)^{10} dx$    i.)  $\int \frac{x+6}{(x+5)^2} dx$    j.)  $\int \frac{(\ln x)^4}{x} dx$

k.)  $\int \sec^2(3x) 2^{\tan(3x)} dx$    l.)  $\int (x+3)\sqrt{x-2} dx$

2.) A three-dimensional solid object lies above the  $x$ -axis from  $x = 0$  to  $x = 4$  centimeters. The cross-sectional area of the solid taken perpendicular to the  $x$ -axis at  $x$  is  $A(x) = 6x^3$  square centimeters. Compute the volume of the solid.

3.) Assume that snow is falling at the rate of  $t + \sqrt{t}$  in./hr. at time  $t$  hours. Determine a definite integral and compute the total amount of snowfall between  $t = 0$  and  $t = 4$  hours.

4.) Use a power  $u$ -substitution to integrate each of the following.

a.)  $\int \frac{1}{1+\sqrt{x}} dx$    b.)  $\int \sqrt{4+\sqrt{x}} dx$    c.)  $\int \frac{\sqrt{x}}{4+x^{1/3}} dx$

5.) a.) The base of a solid lies in the region bounded by the graphs of  $y = 1/x$ ,  $y = x^3$ , and  $x = 2$ . Find the volume of the solid if cross-sections taken perpendicular to the  $x$ -axis at  $x$  are

i.) squares.   ii.) rectangles of height 4.   iii.) semi-circles.

b.) The base of a solid lies in the region bounded by the graphs of  $y = e^x$ ,  $y = 1$ , and  $x = 3$ . Find the volume of the solid if cross-sections taken perpendicular to the  $x$ -axis at  $x$  are

i.) triangles of height 5.   ii.) equilateral triangles.

6.) Find the volume of the solid formed by revolving each region bounded by the given graphs about the given axis.

a.)  $y = x^2 - 1$  and the  $x$ -axis about the  $x$ -axis

b.)  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 4$  about the  $x$ -axis

c.)  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 4$  about the  $y$ -axis

- d.)  $y = 3x$ ,  $y = 6$ , and  $x = 0$  about the  $x$ -axis
- e.)  $y = 2x$ ,  $y = 5 - (1/2)x$ , and  $y = 0$  about the  $y$ -axis
- f.)  $y = x^2$  and  $y = x + 2$  about the line  $y = 4$
- g.)  $y = x^2$  and  $y = x^3$  about the line  $y = 2$
- h.)  $y = x^2$  and  $y = x^3$  about the line  $y = -1$
- i.)  $y = x^2$  and  $y = x^3$  about the line  $x = 3$
- j.)  $y = x^2$  and  $y = x^3$  about the line  $x = -2$

7.) Find the length of each graph on the given interval.

- a.)  $y = x^{3/2}$  on the interval  $[0, 4]$
- b.)  $y = (2/3)(x^2 + 1)^{3/2}$  on the interval  $[0, 2]$
- c.  $y = \frac{x^4}{4} + \frac{1}{8x^2}$  on the interval  $[2, 4]$
- d.)  $y = (1/2)(e^x + e^{-x})$  on the interval  $[0, \ln 2]$

8.) Determine a function having the following properties :

$$f''(x) = 1 + e^{x/2}, f'(0) = -1, \text{ and } f(0) = 3$$

9.) Wildebeests (Gnus) are migratory animals and are an important part of the African ecosystem, since their dung fertilizes the soil and their grazing and trampling encourage new growth along migratory paths. Assume that a herd of wildebeests migrates along a path given by  $y = (1/6)x^3 + \frac{1}{2x}$  from  $x = 1$  to  $x = 50$  miles. Determine the total length of this path.

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THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

10.) Connect 6 toothpicks end-to-end to form 4 triangles.