

Math 17B
 Kouba
 Discussion Sheet 4

1.) Use partial fractions to determine the following indefinite integrals (antiderivatives).

a.) $\int \frac{1}{x^2 - x} dx$ b.) $\int \frac{x + 3}{x^2 - x - 2} dx$ c.) $\int \frac{x^2 + x - 1}{x^2(x + 1)} dx$ d.) $\int \frac{x - 3}{x(x^2 + 1)} dx$

e.) $\int \frac{2x - 1}{x^2(x^2 + 1)} dx$ f.) $\int \frac{x^2 + 4}{x^2(x - 2)^2} dx$ g.) $\int \frac{(x + 1)^2}{(x^2 - x)^2} dx$ h.) $\int \frac{x^2 + 2x}{x^2 - 1} dx$

i.) $\int \frac{1}{x^4 + 1} dx$ (Challenging– Just set up the partial fractions decomposition, but do not solve for the unknown constants.)

2.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\int \frac{(\arctan x)^2}{x^2 + 1} dx$ b.) $\int \frac{1}{x^2 + 1} dx$ c.) $\int \ln x dx$ d.) $\int x(\ln x)^2 dx$

e.) $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx$ f.) $\int \frac{\sqrt{x}}{1 + \sqrt{x}} dx$ g.) $\int \frac{1 + \sqrt{x}}{\sqrt{x}} dx$ h.) $\int \frac{1}{x\sqrt{x-1}} dx$

i.) $\int \frac{2}{x^2 + 4x + 13} dx$ j.) $\int \frac{x^2}{x^2 + 1} dx$ k.) $\int \frac{x^5}{1 + x^6} dx$ l.) $\int \frac{x^2}{1 + x^6} dx$

m.) $\int \arctan x dx$ n.) $\int \frac{e^x}{1 + e^{2x}} dx$ o.) $\int \frac{\cos x}{1 + \sin^2 x} dx$ p.) $\int \frac{3}{x^2 + 4} dx$

q.) $\int \frac{1}{x^2 - 2x + 5} dx$ r.) $\int \frac{1}{x^2 - 6x + 5} dx$ s.) $\int \frac{e^{-x} + 1}{xe^{-x} + 1} dx$ (Challenging)

t.) $\int e^x \cos(e^x) dx$ u.) $\int e^{\sqrt{x}} dx$ v.) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

w.) $\int \sin 2x \cos 2x dx$ x.) $\int \frac{\sin 2x}{\cos 2x} dx$ y.) $\int \sin \sqrt{x} dx$

3.) Find the following antiderivative two ways– using u-substitution with a back substitute and using integration by parts : $\int x^3(1 + x^2)^4 dx$

4.) Find the following antiderivatives by using integration by parts twice, then using algebraic manipulation (integration by parts with a twist).

a.) $\int e^{2x} \sin x \, dx$

b.) $\int \sin x \cos 3x \, dx$

5.) Find the average value of $f(x) = x \ln x$ on the interval $[1, e]$.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

6.) Determine the exact value of the following expression :

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}}$$