Math 17B
Kouba
Discussion Sheet 6

1.) Let S be the amount (pounds) of sugar in a tank at time t (minutes). A solution containing $1/2$ pound of sugar per gallon begins flowing into the tank at the rate of 5 gallons per minute and the well-stirred mixture flows out of the tank at the rate of 5 gallons per minute. Initially, the tank holds 200 gallons with 25 pounds of sugar. Set up a differential equation for the rate $\frac{dS}{dt}$ and solve the D.E. for S. How many pounds of sugar are in the tank after 30 minutes ?

2.) Solve the differential equation for problem 1.) if

a.) the flow rate IN is 5 gallons per minute and the flow rate OUT is 4 gallons per minute.

b.) the flow rate IN is 5 gallons per minute and the flow rate OUT is 7 gallons per minute.

3.) Solve the following first-order linear differential equations.

a.) $y' + y = 3$ b.) $y' - y = x + 1$ c.) $y' + \frac{1}{x}y = e^x - 2$

d.) $y' + 2xy = \frac{2x}{1+e^{x^2}}$ e.) $y' + \frac{1}{x}y = xe^{x^3} + x^2 - 1$

f.) $y' + \tan x \cdot y = \sec x - \tan x + 1$ g.) $y' + \sec x \cdot y = \sec x - \tan x + 1$

h.) $xy' - y = x(\ln x)^2$ i.) $xy' + y = x^3 - x^2 + x - 1$ j.) $x^2y' + 2xy = x \ln x$

4.) Use any method to solve the following differential equations. Some can be integrated directly. Some may be first-order linear or separable.

a.) $\frac{dy}{dx} = \ln x$ b.) $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$ c.) $\frac{dy}{dx} = \frac{y^2}{x(1+y^3)}$

d.) $\frac{dy}{dx} - x^5y^2 = 0$ e.) $\frac{dy}{dx} = \frac{x+x^3}{1+x^4}$ f.) $\frac{dy}{dx} - y^2 = y$

g.) $\frac{dy}{dx} + 3x^2y = 7x^2$ h.) $\frac{dy}{dx} = \frac{x\cos(x^2)}{y^2 \sin y}$ i.) $y' + y^3 = y$ and $x = 0, y = 2$

j.) $y' + 2y = e^{-2x}\tan^2(7x)$ k.) $y' \cdot \cos^2 x + y = 1$ l.) $xy' + 2y = x\cos x$

m.) $\tan x \cdot y' = y(y+1)\cot x$ n.) $\cos(5x^2) \cdot y' = x \cdot \sec^2(3y)$

o.) $e^y \cdot \sin(e^y) \cdot y' = (1+e^x)e^x$ p.) $\cos^3 y \cdot \sin y \cdot dy = \tan^3(10x) \cdot dx$

5.) Consider the differential equation $\frac{dy}{dx} + 3x = 2xy$.

a.) Write it in separable form and solve it.

b.) Write it in first-order linear form and solve it.

6.) Determine the equilibia for each autonomous D.E. and determine their stability using the indicated method.

a.) $\frac{dN}{dt} = N^3 - 4N$ (Sign Chart Method)

b.) $\frac{dN}{dt} = N(3-N)(N-5)$ (Derivative, $g'(N)$, Method)

c.) $\frac{dN}{dt} = \frac{N^2 - 9N}{N^2 + 9}$ (Any Method)

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THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

7.) A camp cook wants to measure four ounces of vinegar out of a jug, but he has only an unmarked five-ounce container and an unmarked three-ounce container. How can he do it?