

Math 17B  
Kouba  
Discussion Sheet 7

1.) Let  $S$  be the amount (pounds) of sugar in a tank at time  $t$  (minutes). A solution containing  $1/2$  pound of sugar per gallon begins flowing into the tank at the rate of 5 gallons per minute and the well-stirred mixture flows out of the tank at the rate of 5 gallons per minute. Initially, the tank holds 200 gallons with 25 pounds of sugar. Set up a differential equation for the rate  $\frac{dS}{dt}$  and solve the D.E. for  $S$ . How many pounds of sugar are in the tank after 30 minutes?

2.) Solve the differential equation for problem 1.) if

a.) the flow rate IN is 5 gallons per minute and the flow rate OUT is 4 gallons per minute.

b.) the flow rate IN is 5 gallons per minute and the flow rate OUT is 7 gallons per minute.

3.) Determine the equilibria for each autonomous D.E. and determine their stability using the indicated method.

a.)  $\frac{dN}{dt} = N^3 - 4N$  (Sign Chart Method)

b.)  $\frac{dN}{dt} = N(3 - N)(N - 5)$  (Derivative,  $g'(N)$ , Method)

c.)  $\frac{dN}{dt} = \frac{N^2 - 9N}{N^2 + 9}$  (Any Method)

4.) Use any method to solve the following differential equations. Some can be integrated directly. Some may be first-order linear or separable.

a.)  $\frac{dy}{dx} = \ln x$     b.)  $\frac{dy}{dx} = \frac{x^2}{y(1 + x^3)}$     c.)  $\frac{dy}{dx} = \frac{y^2}{x(1 + y^3)}$

d.)  $\frac{dy}{dx} - x^5 y^2 = 0$     e.)  $\frac{dy}{dx} = \frac{x + x^3}{1 + x^4}$     f.)  $\frac{dy}{dx} - y^2 = y$

g.)  $\frac{dy}{dx} + 3x^2 y = 7x^2$     h.)  $\frac{dy}{dx} = \frac{x \cos(x^2)}{y^2 \sin y}$

5.) Consider the differential equation  $\frac{dy}{dx} + 3x = 2xy$ .

a.) Write it in separable form and solve it.

b.) Write it in first-order linear form and solve it.

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THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

6.) A camp cook wants to measure four ounces of vinegar out of a jug, but he has only an unmarked five-ounce container and an unmarked three-ounce container. How can he do it?