Math 17B
Kouba
Discussion Sheet 9

1.) Consider the following matrices of various dimensions. Perform the indicated operations.

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & -2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & -1 & 0 & 4 \end{pmatrix}, \quad F = \begin{pmatrix} 2 \\ -2 \\ 3 \\ -1 \end{pmatrix}$$

a.) $3C$ b.) $A - B$ c.) B', the transpose of B

d.) CD e.) AB f.) BA g.) EF h.) FE

i.) Compute C^{-1}, the inverse of C j.) Compute A^{-1}, the inverse of A.

2.) Consider the system $\begin{cases} x - y = 3 \\ 2x + 3y = 26 \end{cases}$

a.) Solve the system using matrix reduction.
b.) Rewrite the system in the form $AX = B$ and solve it using A^{-1}.

3.) Compute the determinant of each of the following matrices to determine which matrices are invertible. If the matrix is invertible, determine its inverse matrix.

a.) $A = \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}$ b.) $A = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ c.) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ d.) $A = \begin{pmatrix} 0 & 0 \\ 3 & 5 \end{pmatrix}$

4.) Determine the inverse for each $3x3$ matrix.

a.) $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 1 \\ 0 & 1 & -2 \end{pmatrix}$ b.) $A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 1 & -1 \\ 3 & -2 & 1 \end{pmatrix}$ c.) $A = \begin{pmatrix} 3 & 1 & 4 \\ 4 & 1 & 6 \\ 1 & 0 & 1 \end{pmatrix}$

5.) Use your answer in part 4.)a.) to solve the system $\begin{cases} x + 3z = 2 \\ 2x - y + z = -3 \\ y - 2z = 0 \end{cases}$

6.) Let $A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 0 \\ -1 & 4 \end{pmatrix}$ Show that AB is not equal to BA.

7.) Let $A = \begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix}$.

a.) Find a 2x2 matrix B so that $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, or explain why it is impossible.

b.) Find a 2x2 matrix B, where NOT ALL of the entries are zero, so that $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, or explain why it is impossible.

8.) Consider the Leslie matrix $L = \begin{pmatrix} 0 & 1.4 & 1.8 & 2.7 & 0.9 \\ 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 \end{pmatrix}$

a.) How many age classes are in this population ?

b.) What percenatge of 2-year old females survive to the end of the following breeding season ?

c.) What percenatge of 4-year old females survive to the end of the following breeding season ?

d.) What is an average number of female offspring for a 0-year old female ?

e.) What is an average number of female offspring for a 1-year old female ?

f.) What is an average number of female offspring for a 3-year old female ?

g.) If $N(0) = \begin{pmatrix} 1000 \\ 650 \\ 450 \\ 550 \\ 300 \end{pmatrix}$, determine $N(1)$.

" I hear and I forget. I see and I remember. I do and I understand."– Chinese Proverb