

Integration by Parts

RECALL: (Product Rule)

$$D \{f(x)g(x)\} = f'(x)g(x) + f(x)g'(x)$$

Then

$$\int D \{f(x)g(x)\} dx = \int g(x)f'(x) dx + \int f(x)g'(x) dx$$

$$\left(\begin{array}{l} \text{Let } u = f(x) \xrightarrow{D} du = f'(x) dx \text{ and} \\ \text{let } v = g(x) \xrightarrow{D} dv = g'(x) dx \end{array} \right) \rightarrow$$

$$f(x)g(x) = \int v du + \int u dv \rightarrow$$

$$uv = \int v du + \int u dv \rightarrow$$

$$\boxed{\int u dv = uv - \int v du}$$

$$\underline{\text{Ex}}: \int x e^{2x} dx \quad \left(\begin{array}{l} \text{Let } u = x, dv = e^{2x} dx \\ \rightarrow du = 1 \cdot dx, v = \frac{1}{2} e^{2x} \end{array} \right)$$

$$= x \cdot \left(\frac{1}{2} e^{2x} \right) - \int \frac{1}{2} e^{2x} dx$$

$$= \frac{1}{2} x e^{2x} - \frac{1}{2} \cdot \frac{1}{2} e^{2x} + c$$