Linear Systems of Equations

Ex: Raw almonds selling for \$5/lb. are mixed with raw almonds selling for \$2/lb. How many pounds of each should be mixed to result in 100 pounds of almonds selling for \$4/lb.?

Let x: lbs. of \$5 almonds
y: lbs. of \$2 almonds

then

(weight): $x + y = 100$ (lbs.)

(total cost): $\left(\frac{\$5}{lb.}\right)(x\ lbs.) + \left(\frac{\$2}{lb.}\right)(y\ lbs.) = \left(\frac{\$4}{lb.}\right)(100\ lbs.)$

$$\rightarrow \begin{cases} x + y = 100 \\ 5x + 2y = 400 \end{cases}$$

$$\rightarrow \begin{cases} -5x + -5y = -500 \\ 5x + 2y = 400 \end{cases}$$

$$\rightarrow -3y = -100 \rightarrow y = \frac{100}{3}\ lbs.,$$

$$x = \frac{200}{3}\ lbs.$$

Solving Two Equations, Two Unknowns

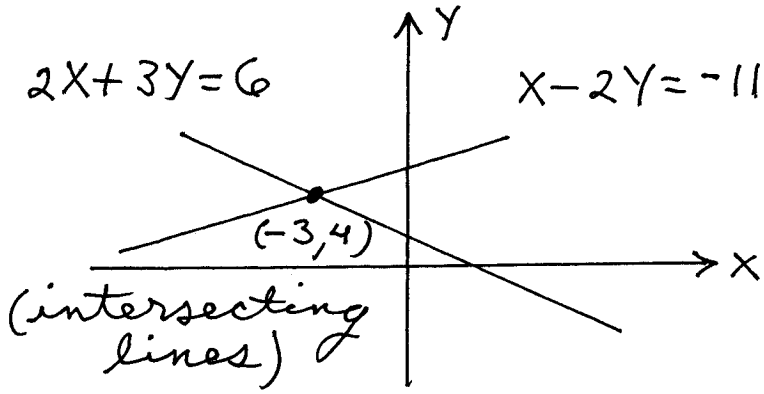
Ex: Solve the following systems.

1.) $\left.\begin{aligned} x - 2y &= -11 \\ 2x + 3y &= 6 \end{aligned}\right\} \rightarrow \left.\begin{aligned} -2x + 4y &= 22 \\ 2x + 3y &= 6 \end{aligned}\right\}$

$\rightarrow 7y = 28$

$\rightarrow$ $(y = 4)$, $(x = -3)$

(ONE SOLUTION)

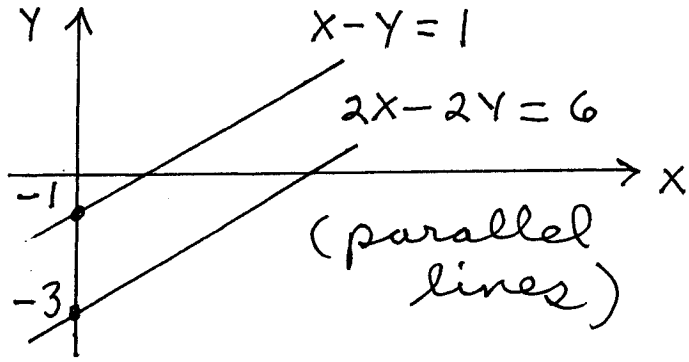


2.) $\left.\begin{aligned} x - y &= 1 \\ 2x - 2y &= 6 \end{aligned}\right\} \rightarrow \left.\begin{aligned} -2x + 2y &= -2 \\ 2x - 2y &= 6 \end{aligned}\right\}$

$\rightarrow 0 = 4$!!!

IMPOSSIBLE so

(NO SOLUTION)



3.) $\left.\begin{aligned} x + 3y &= -2 \\ -2x - 6y &= 4 \end{aligned}\right\} \rightarrow \left.\begin{aligned} 2x + 6y &= -4 \\ -2x - 6y &= 4 \end{aligned}\right\}$

$\rightarrow 0 = 0$ (REDUNDANT); note that

$\left.\begin{aligned} x + 3y &= -2 \\ x + 3y &= -2 \end{aligned}\right\}$ (OVERDETERMINED SYSTEM)

Then $x + 3y = -2 \rightarrow$

$x = -3y - 2$, so let $\boxed{y = t \text{ any } \#}$

and $\boxed{x = -3t - 2}$;

INFINITELY MANY SOLUTIONS