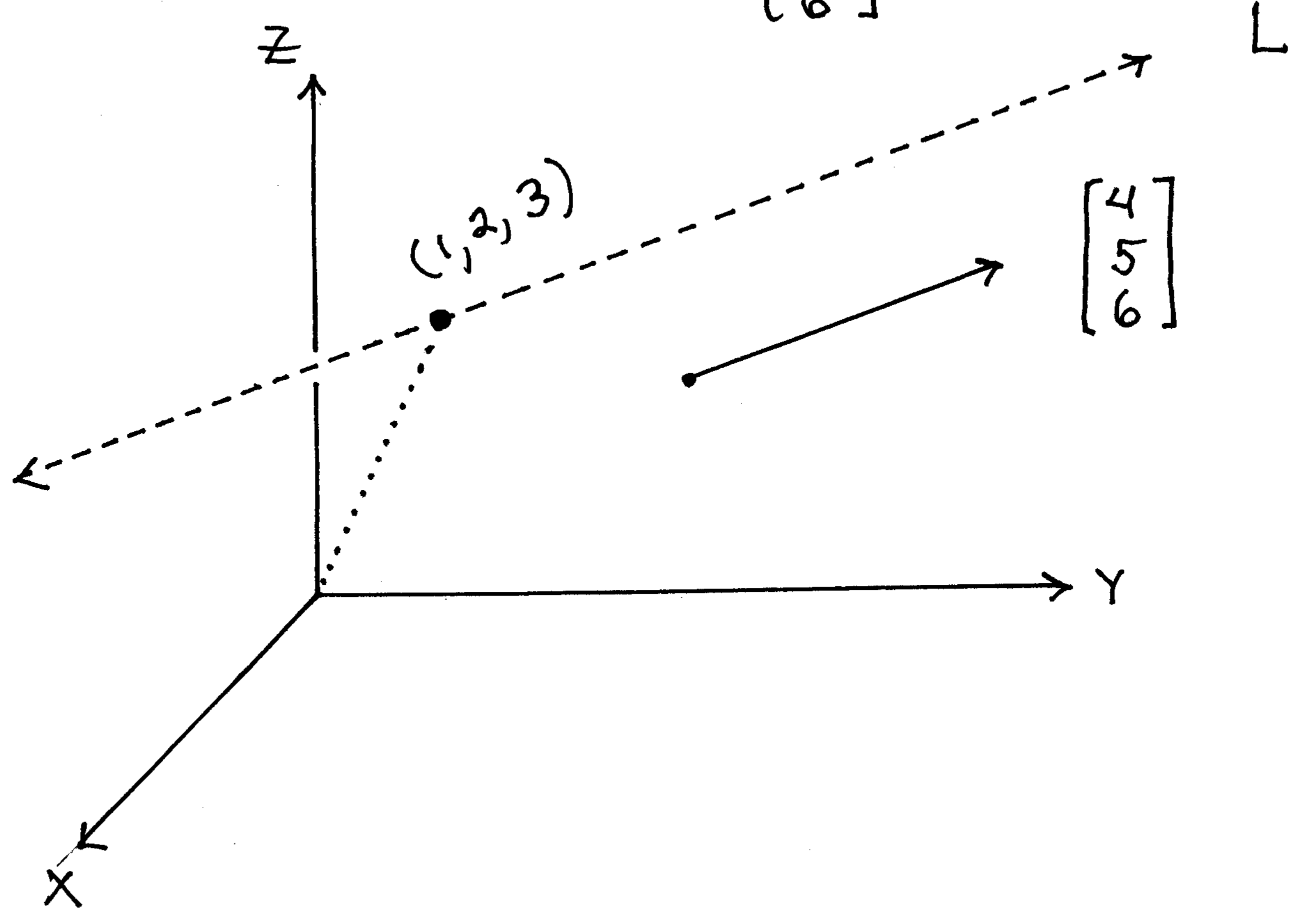
Creating Representations (Vector Form and Parametric Form) for a Line in $\mathbb{R}^3$

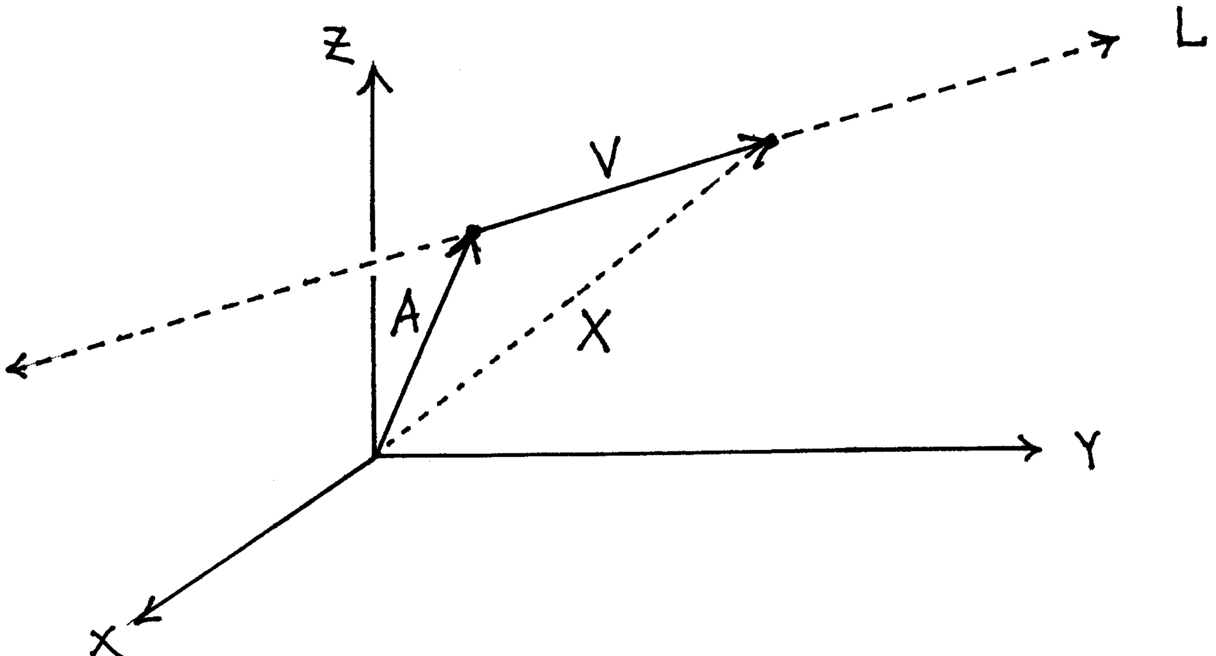
Example: Determine the parametric form of a line L passing through point (1, 2, 3) and parallel to (in the direction of) vector $V = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$:



Start by forming vector A from point (0,0,0) to point (1,2,3), i.e.,

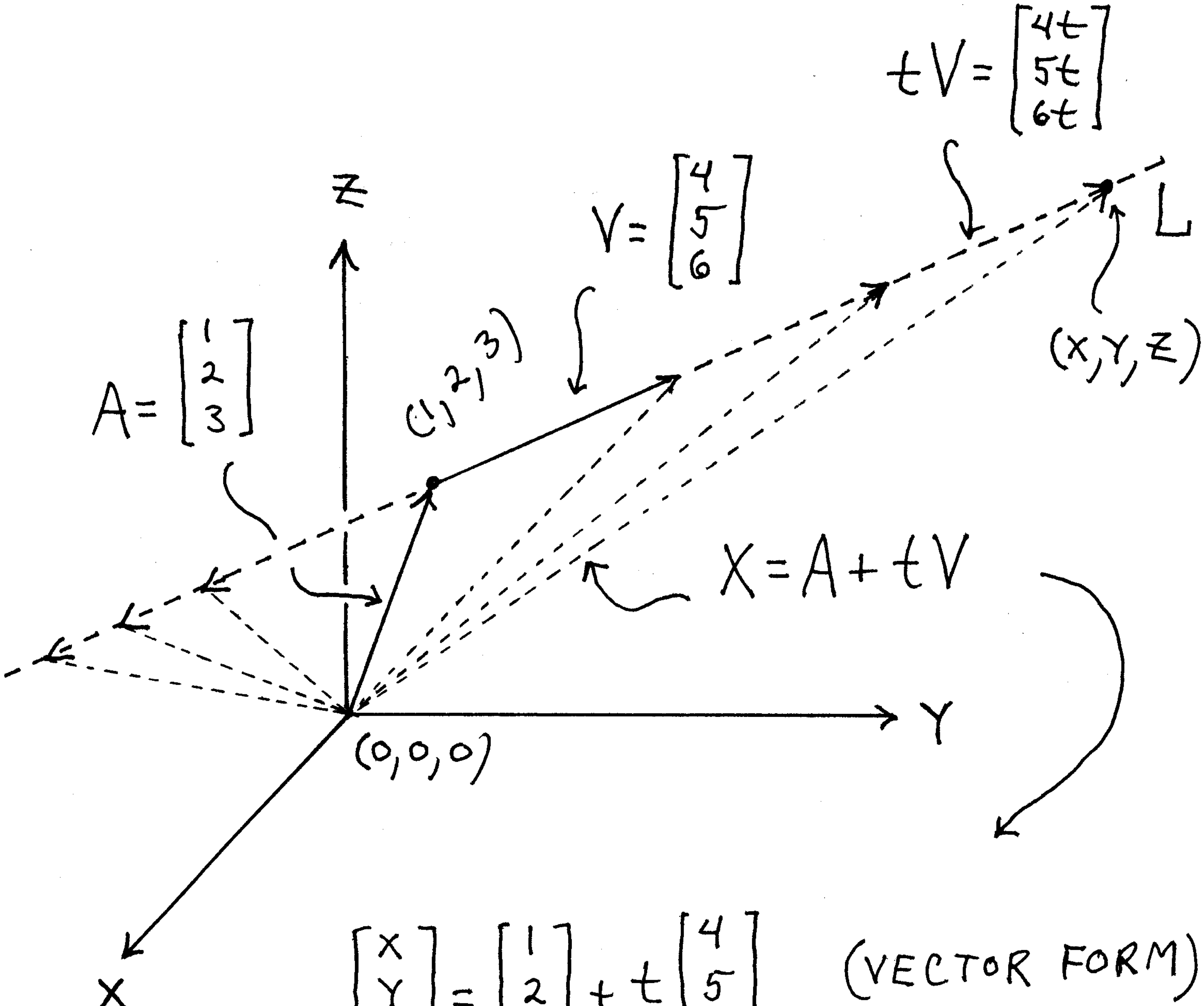
$$A = \begin{bmatrix} 1-0 \\ 2-0 \\ 3-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},$$ and attaching vector

V to the tip of vector A. Now consider the vector $X = A + V$:



Now consider multiples tV of V and let (X, Y, Z) be a random variable point at the tip of vector tV. Form vector X from point (0,0,0) to point (X, Y, Z), i.e.,

$$X = \begin{bmatrix} X-0 \\ Y-0 \\ Z-0 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}.$$ We have that :



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad \text{(VECTOR FORM)}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4t \\ 5t \\ 6t \end{bmatrix} = \begin{bmatrix} 1+4t \\ 2+5t \\ 3+6t \end{bmatrix} \longrightarrow$$

$$L : \begin{cases} x = 1 + 4t \\ y = 2 + 5t \\ z = 3 + 6t \end{cases} \quad \text{(PARAMETRIC FORM)}$$