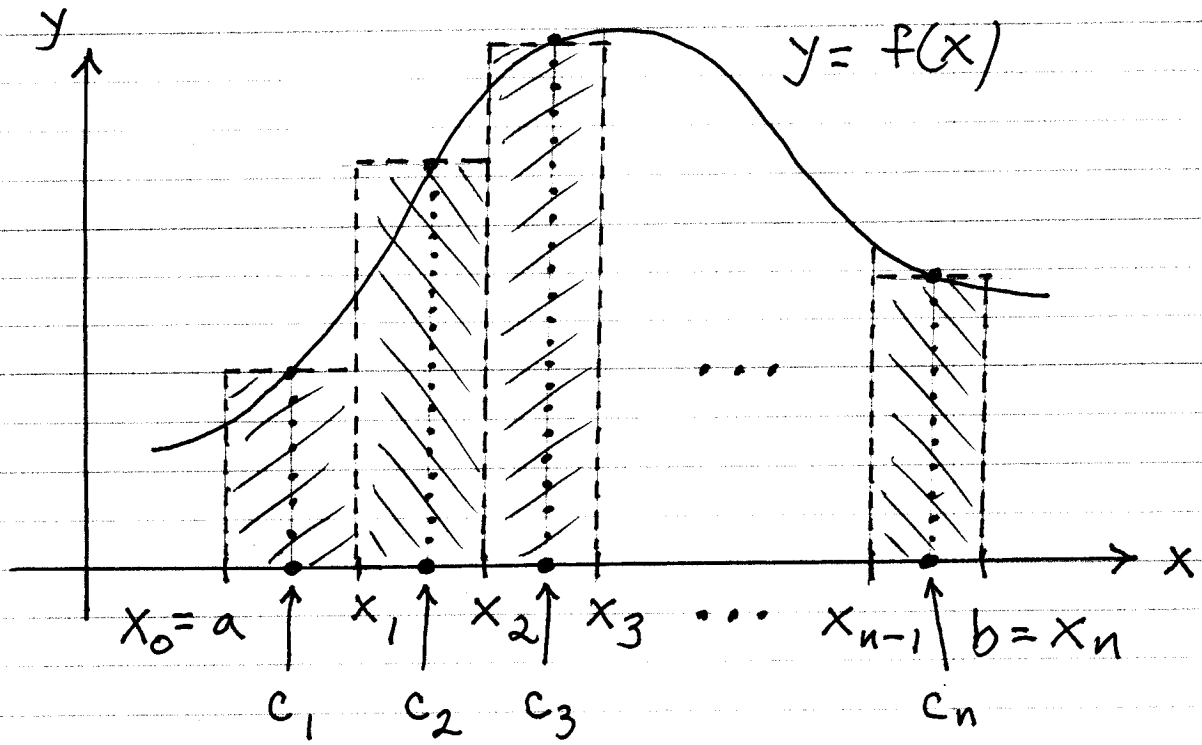
I.) Midpoint Rule

Divide interval $[a,b]$ into n equal parts each of length

$$h = \frac{b-a}{n}.$$

Let $c_1, c_2, c_3, \ldots c_n$ be the midpoints of the subintervals. Form rectangles:



Then

$$\int_a^b f(x)\,dx \approx h\cdot f(c_1) + h\cdot f(c_2) + h\cdot f(c_3) + h\cdot f(c_{n-1}) + h\cdot f(c_n)$$

$$= h\left[f(c_1) + f(c_2) + f(c_3) + \cdots + f(c_n)\right]$$

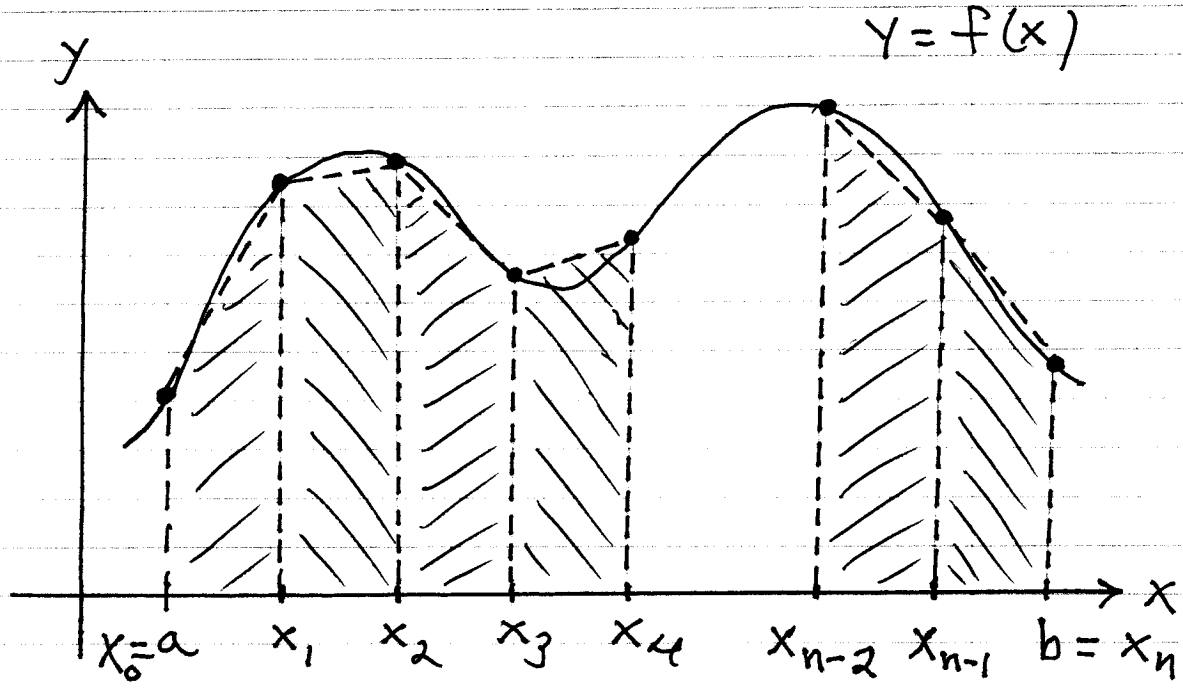
$$= M_n$$

II. Trapezoidal Rule

Divide interval $[a,b]$ into n equal parts each of length

$$h = \frac{b-a}{n}.$$

Form trapezoids:



Then

$$\int_a^b f(x)\,dx \approx h\cdot\frac{1}{2}(f(x_0)+f(x_1)) + h\cdot\frac{1}{2}(f(x_1)+f(x_2))$$

$$+ h\cdot\frac{1}{2}(f(x_2)+f(x_3)) + \cdots + h\cdot\frac{1}{2}(f(x_{n-2})+f(x_{n-1}))$$

$$+ h\cdot\frac{1}{2}(f(x_{n-1}) + f(x_n))$$

$$= \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

$$= T_n$$